any two contiguous functions have been given by Gauss. By repeated application of these relations the function $F(a+m, b+n; c+l; z)$ with integral m, n, $l (c+l \neq 0, -1, -2, \ldots)$ can be expressed as a linear combination of $F(a, b; c; z)$ and one of its contiguous functions with coefficients which are rational functions of a, b, c, z.

15.2.10

$$(c-a)F(a-1, b; c; z)+(2a-c-az+bz)F(a, b; c; z) +a(z-1)F(a+1, b; c; z)=0$$

15.2.11

$$(c-b)F(a, b-1; c; z)+(2b-c-bz+az)F(a, b; c; z) +b(z-1)F(a, b+1; c; z)=0$$

15.2.12

$$c(c-1)(z-1)F(a, b; c-1; z) +c[c-1-(2c-a-b-1)z]F(a, b; c; z) +(c-a)(c-b)zF(a, b; c+1; z)=0$$

15.2.13

$$[c-2a-(b-a)z]F(a, b; c; z) +a(1-z)F(a+1, b; c; z) -(c-a)F(a-1, b; c; z)=0$$

15.2.14

$$(b-a)F(a, b; c; z)+aF(a+1, b; c; z) -bF(a, b+1; c; z)=0$$

15.2.15

$$(c-a-b)F(a, b; c; z)+a(1-z)F(a+1, b; c; z) -(c-b)F(a, b-1; c; z)=0$$

15.2.16

$$c[a-(c-b)z]F(a, b; c; z)-ac(1-z)F(a+1, b; c; z) +(c-a)(c-b)zF(a, b; c+1; z)=0$$

15.2.17

$$(c-a-1)F(a, b; c; z)+aF(a+1, b; c; z) -(c-1)F(a, b; c-1; z)=0$$

15.2.18

$$(c-a-b)F(a, b; c; z)-(c-a)F(a-1, b; c; z) +b(1-z)F(a, b+1; c; z)=0$$

15.2.19

$$(b-a)(1-z)F(a, b; c; z)-(c-a)F(a-1, b; c; z) +(c-b)F(a, b-1; c; z)=0$$

15.2.20

$$c(1-z)F(a, b; c; z)-cF(a-1, b; c; z) +(c-b)zF(a, b; c+1; z)=0$$

15.2.21

$$[a-1-(c-b-1)z]F(a, b; c; z) +(c-a)F(a-1, b; c; z) -(c-1)(1-z)F(a, b; c-1; z)=0$$

15.2.22

$$[c-2b+(b-a)z]F(a, b; c; z) +b(1-z)F(a, b+1; c; z) -(c-b)F(a, b-1; c; z)=0$$

15.2.23

$$c[b-(c-a)z]F(a, b; c; z)-bc(1-z)F(a, b+1; c; z) +(c-a)(c-b)zF(a, b; c+1; z)=0$$

15.2.24

$$(c-b-1)F(a, b; c; z)+bF(a, b+1; c; z) -(c-1)F(a, b; c-1; z)=0$$

15.2.25

$$c(1-z)F(a, b; c; z)-cF(a, b-1; c; z) {}^{*}+(c-a)zF(a, b; c+1; z)=0$$

15.2.26

$$[b-1-(c-a-1)z]F(a, b; c; z) +(c-b)F(a, b-1; c; z) -(c-1)(1-z)F(a, b; c-1; z)=0$$

15.2.27

$$c[c-1-(2c-a-b-1)z]F(a, b; c; z) +(c-a)(c-b)zF(a, b; c+1; z) -c(c-1)(1-z)F(a, b; c-1; z)=0$$

15.3. Integral Representations and Transformation Formulas

Integral Representations

15.3.1

$$F(a, b; c; z)= \frac{\Gamma(c)}{\Gamma(b)\Gamma(c-b)} \int_0^1 t^{b-1}(1-t)^{c-b-1}(1-tz)^{-a}dt \qquad (\mathscr{R}c>\mathscr{R}b>0)$$

The integral represents a one valued analytic function in the z-plane cut along the real axis from 1 to ∞ and hence **15.3.1** gives the analytic continuation of **15.1.1**, $F(a, b; c; z)$. Another integral representation is in the form of a Mellin-Barnes integral

*See page II.