13.6. Special Cases—Continued

| | $U(a, b, z)$ a | $U(a, b, z)$ b | $U(a, b, z)$ z | Relation | Function |
|---|---|---|---|---|---|
| 13.6.21 | $\nu+\frac{1}{2}$ | $2\nu+1$ | $2z$ | $\pi^{-\frac{1}{2}}e^{z}(2z)^{-\nu}K_{\nu}(z)$ | Modified Bessel |
| 13.6.22 | $\nu+\frac{1}{2}$ | $2\nu+1$ | $-2iz$ | $\frac{1}{2}\pi^{\frac{1}{2}}e^{i[\pi(\nu+\frac{1}{2})-z]}(2z)^{-\nu}H_{\nu}^{(1)}(z)^{*}$ | Hankel |
| 13.6.23 | $\nu+\frac{1}{2}$ | $2\nu+1$ | $2iz$ | $\frac{1}{2}\pi^{\frac{1}{2}}e^{-i[\pi(\nu+\frac{1}{2})-z]}(2z)^{-\nu}H_{\nu}^{(2)}(z)^{*}$ | Hankel |
| 13.6.24 | $n+1$ | $2n+2$ | $2z$ | $\pi^{-\frac{1}{2}}e^{z}(2z)^{-n-\frac{1}{2}}K_{n+\frac{1}{2}}(z)$ | Spherical Bessel |
| 13.6.25 | $\frac{5}{6}$ | $\frac{5}{3}$ | $\frac{4}{3}z^{3/2}$ | $\pi^{\frac{1}{2}}z^{-1}\exp(\frac{2}{3}z^{3/2})2^{-2/3}3^{5/6}\,\mathrm{Ai}\,(z)$ | Airy |
| 13.6.26 | $n+\frac{1}{2}$ | $2n+1$ | $\sqrt{ix}$ | $i^{n}\pi^{-\frac{1}{2}}e^{\sqrt{ix}}(2\sqrt{ix})^{-n}[\mathrm{ker}_n\, x+i\,\mathrm{kei}_n\, x]$ | Kelvin |
| 13.6.27 | $-n$ | $\alpha+1$ | $x$ | $(-1)^{n}n!L_{n}^{(\alpha)}(x)$ | Laguerre |
| 13.6.28 | $1-a$ | $1-a$ | $x$ | $e^{x}\Gamma(a, x)$ | Incomplete Gamma |
| 13.6.29 | 1 | 1 | $-x$ | $-e^{-x}\,\mathrm{Ei}\,(x)$ | Exponential Integral |
| 13.6.30 | 1 | 1 | $x$ | $e^{x}E_1(x)$ | Exponential Integral |
| 13.6.31 | 1 | 1 | $-\ln x$ | $-\frac{1}{x}\,\mathrm{li}\,(x)$ | Logarithmic Integral |
| 13.6.32 | $\frac{1}{2}m-n$ | $1+m$ | $x$ | $\Gamma(1+n-\frac{1}{2}m)e^{x-\pi i(\frac{1}{2}m-n)}\omega_{n,m}(x)$ | Cunningham |
| 13.6.33 | $-\frac{1}{2}\nu$ | 0 | $2x$ | $\Gamma(1+\frac{1}{2}\nu)e^{x}k_{\nu}(x)$ for $x>0$ | Bateman |
| 13.6.34 | 1 | 1 | $ix$ | $e^{ix}[-\frac{1}{2}\pi i+i\,\mathrm{Si}\,(x)-\mathrm{Ci}\,(x)]$ | Sine and Cosine Integral |
| 13.6.35 | 1 | 1 | $-ix$ | $e^{-ix}[\frac{1}{2}\pi i-i\,\mathrm{Si}\,(x)-\mathrm{Ci}\,(x)]$ | Sine and Cosine Integral |
| 13.6.36 | $-\frac{1}{2}\nu$ | $\frac{1}{2}$ | $\frac{1}{2}z^2$ | $2^{-\frac{1}{2}\nu}e^{z^2/4}D_{\nu}(z)$ | Weber or |
| 13.6.37 | $\frac{1}{2}-\frac{1}{2}\nu$ | $\frac{3}{2}$ | $\frac{1}{2}z^2$ | $2^{\frac{1}{2}-\frac{1}{2}\nu}e^{z^2/4}D_{\nu}(z)/z$ * | Parabolic Cylinder |
| 13.6.38 | $\frac{1}{2}-\frac{1}{2}n$ | $\frac{3}{2}$ | $x^2$ | $2^{-n}H_n(x)/x$ * | Hermite |
| 13.6.39 | $\frac{1}{2}$ | $\frac{1}{2}$ | $x^2$ | $\sqrt{\pi}\exp(x^2)\,\mathrm{erfc}\,x$ | Error Integral |

## 13.7. Zeros and Turning Values

If $j_{b-1,r}$ is the $r$'th positive zero of $J_{b-1}(x)$, then a first approximation $X_0$ to the $r$'th positive zero of $M(a, b, x)$ is

**13.7.1** $$X_0=j^2_{b-1,r}\{1/(2b-4a)+O(1/(\tfrac{1}{2}b-a)^2)\}$$

**13.7.2** $$X_0\approx\frac{\pi^2(r+\frac{1}{2}b-\frac{3}{4})^2}{2b-4a}$$

A closer approximation is given by

**13.7.3** $$X_1=X_0-M(a,b,X_0)/M'(a,b,X_0)$$

For the derivative,

**13.7.4**

$$M'(a,b,X_1)=M'(a,b,X_0)\left\{1+(b-X_0)\frac{M(a,b,X_0)}{M'(a,b,X_0)}\right\}$$

If $X_0'$ is the first approximation to a turning value of $M(a, b, x)$, that is, to a zero of $M'(a, b, x)$ then a better approximation is

**13.7.5** $$X_1'=X_0'-\frac{X_0'M'(a,b,X_0')}{aM(a,b,X_0')}$$

*See page II.