13.1.15 $y_4=z^{1-b}e^zM(1-a, 2-b, -z)$

13.1.16 $y_5=U(a, b, z)$

13.1.17 $y_6=z^{1-b}U(1+a-b, 2-b, z)$

13.1.18 $y_7=e^zU(b-a, b, -z)$

13.1.19 $y_8=z^{1-b}e^zU(1-a, 2-b, -z)$

Wronskians

If $W\{m, n\}=y_my_n'-y_ny_m'$ and
$\epsilon=sgn\ (\mathscr{I}z)=1$ if $\mathscr{I}z>0$,
$=-1$ if $\mathscr{I}z\leq 0$

13.1.20

$$W\{1, 2\}=W\{3, 4\}=W\{1, 4\}=-W\{2, 3\}=(1-b)z^{-b}e^z$$

13.1.21

$$W\{1, 3\}=W\{2, 4\}=W\{5, 6\}=W\{7, 8\}=0$$

13.1.22 $W\{1, 5\}=-\Gamma(b)z^{-b}e^z/\Gamma(a)$

13.1.23 $W\{1, 7\}=\Gamma(b)e^{\epsilon\pi ib}z^{-b}e^z/\Gamma(b-a)$

13.1.24 $W\{2, 5\}=-\Gamma(2-b)z^{-b}e^z/\Gamma(1+a-b)$

13.1.25 $W\{2, 7\}=-\Gamma(2-b)z^{-b}e^z/\Gamma(1-a)$

13.1.26 $W\{5, 7\}=e^{\epsilon\pi i(b-a)}z^{-b}e^z$

Kummer Transformations

13.1.27 $M(a, b, z)=e^zM(b-a, b, -z)$

13.1.28

$$z^{1-b}M(1+a-b, 2-b, z)=z^{1-b}e^zM(1-a, 2-b, -z)$$

13.1.29 $U(a, b, z)=z^{1-b}U(1+a-b, 2-b, z)$

13.1.30

$$e^zU(b-a, b, -z)=e^{\epsilon\pi i(1-b)}e^zz^{1-b}U(1-a, 2-b, -z)$$

Whittaker's Equation

13.1.31 $\dfrac{d^2w}{dz^2}+\left[-\dfrac{1}{4}+\dfrac{\kappa}{z}+\dfrac{(\frac{1}{4}-\mu^2)}{z^2}\right]w=0$

Solutions:

Whittaker's Functions

13.1.32 $M_{\kappa,\mu}(z)=e^{-\frac{1}{2}z}z^{\frac{1}{2}+\mu}M(\frac{1}{2}+\mu-\kappa, 1+2\mu, z)$

13.1.33

$$W_{\kappa,\mu}(z)=e^{-\frac{1}{2}z}z^{\frac{1}{2}+\mu}U(\tfrac{1}{2}+\mu-\kappa, 1+2\mu, z)$$
$$(-\pi<\arg z\leq\pi,\ \kappa=\tfrac{1}{2}b-a,\ \mu=\tfrac{1}{2}b-\tfrac{1}{2})$$

13.1.34

$$W_{\kappa,\mu}(z)=\frac{\Gamma(-2\mu)}{\Gamma(\frac{1}{2}-\mu-\kappa)}M_{\kappa,\mu}(z)+\frac{\Gamma(2\mu)}{\Gamma(\frac{1}{2}+\mu-\kappa)}M_{\kappa,-\mu}(z)$$

General Confluent Equation

13.1.35

$$w''+\left[\frac{2A}{Z}+2f'+\frac{bh'}{h}-h'-\frac{h''}{h'}\right]w'+\left[\left(\frac{bh'}{h}-h'-\frac{h''}{h'}\right)\left(\frac{A}{Z}+f'\right)+\frac{A(A-1)}{Z^2}+\frac{2Af'}{Z}+f''+f'^2-\frac{ah'^2}{h}\right]w=0$$

Solutions:

13.1.36 $Z^{-A}e^{-f(Z)}M(a, b, h(Z))$

13.1.37 $Z^{-A}e^{-f(Z)}U(a, b, h(Z))$

13.2. Integral Representations

$$\mathscr{R}b>\mathscr{R}a>0$$

13.2.1

$$\frac{\Gamma(b-a)\Gamma(a)}{\Gamma(b)}M(a, b, z)=\int_0^1 e^{zt}t^{a-1}(1-t)^{b-a-1}dt$$

13.2.2

$$=2^{1-b}e^{\frac{1}{2}z}\int_{-1}^{+1}e^{-\frac{1}{2}zt}(1+t)^{b-a-1}(1-t)^{a-1}dt$$

13.2.3

$$=2^{1-b}e^{\frac{1}{2}z}\int_0^\pi e^{-\frac{1}{2}z\cos\theta}\sin^{b-1}\theta\cot^{b-2a}(\tfrac{1}{2}\theta)\,d\theta$$

13.2.4

$$=e^{-Az}\int_A^B e^{zt}(t-A)^{a-1}(B-t)^{b-a-1}dt \qquad (A=B-1)$$
$$\mathscr{R}a>0,\ \mathscr{R}z>0$$

13.2.5

$$\Gamma(a)U(a, b, z)=\int_0^\infty e^{-zt}t^{a-1}(1+t)^{b-a-1}dt$$

13.2.6

$$=e^z\int_1^\infty e^{-zt}(t-1)^{a-1}t^{b-a-1}dt$$

13.2.7

$$=2^{1-b}e^{\frac{1}{2}z}\int_0^\infty e^{-\frac{1}{2}z\cosh\theta}\sinh^{b-1}\theta\coth^{b-2a}(\tfrac{1}{2}\theta)d\theta$$ *

*See page II.