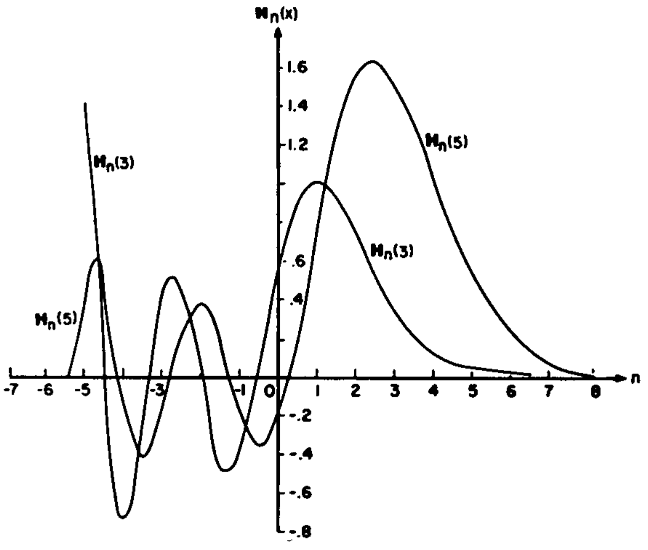


FIGURE 12.3. *Struve functions.*

$\mathbf{H}_n(x)$, $x=3, 5$

### Special Properties

**12.1.14** $\mathbf{H}_\nu(x)\geq 0$ $\quad(x>0 \text{ and } \nu\geq\frac{1}{2})$

**12.1.15**

$$\mathbf{H}_{-(n+\frac{1}{2})}(z)=(-1)^n J_{n+\frac{1}{2}}(z)\quad(n \text{ an integer}\geq 0)$$

**12.1.16** $$\mathbf{H}_{\frac{1}{2}}(z)=\left(\frac{2}{\pi z}\right)^{\frac{1}{2}}(1-\cos z)$$

**12.1.17**

$$\mathbf{H}_{\frac{3}{2}}(z)=\left(\frac{z}{2\pi}\right)^{\frac{1}{2}}\left(1+\frac{2}{z^2}\right)-\left(\frac{2}{\pi z}\right)^{\frac{1}{2}}\left(\sin z+\frac{\cos z}{z}\right)$$

**12.1.18** $\mathbf{H}_\nu(ze^{m\pi i})=e^{m(\nu+1)\pi i}\,\mathbf{H}_\nu(z)$ ($m$ an integer)

**12.1.19** $$\mathbf{H}_0(z)=\frac{4}{\pi}\sum_{k=0}^{\infty}\frac{J_{2k+1}(z)}{2k+1}$$

**12.1.20** $$\mathbf{H}_1(z)=\frac{2}{\pi}-\frac{2}{\pi}J_0(z)+\frac{4}{\pi}\sum_{k=1}^{\infty}\frac{J_{2k}(z)}{4k^2-1}$$

**12.1.21** $$\mathbf{H}_\nu(z)=\frac{2(z/2)^{\nu+1}}{\sqrt{\pi}\,\Gamma(\nu+\frac{3}{2})}\,{}_1F_2\left(1;\frac{3}{2}+\nu,\frac{3}{2};-\frac{z^2}{4}\right)$$

### Integrals (See chapter 11)

**12.1.22** $$\int_0^\infty t^{-1}\,\mathbf{H}_0(t)dt=\frac{\pi}{2}$$

**12.1.23**

$$\int_0^z \mathbf{H}_0(t)dt=\frac{2}{\pi}\left[\frac{z^2}{2}-\frac{z^4}{1^2\cdot 3^2\cdot 4}+\frac{z^6}{1^2\cdot 3^2\cdot 5^2\cdot 6}-\dots\right]$$

**12.1.24** $$\int_0^z t^{-\nu}\,\mathbf{H}_{\nu+1}(t)dt=\frac{z}{2^\nu\sqrt{\pi}\,\Gamma(\nu+\frac{3}{2})}-z^{-\nu}\,\mathbf{H}_\nu(z)$$

### Struve's Integral

**12.1.25**

$$\frac{4}{\pi}\int_z^\infty t^{-2}\,\mathbf{H}_1(t)dt=\frac{2}{\pi z}\mathbf{H}_1(z)+\frac{2}{\pi}\int_z^\infty t^{-1}\,\mathbf{H}_0(t)dt$$

**12.1.26**

$$\frac{2}{\pi}\int_z^\infty t^{-1}\,\mathbf{H}_0(t)dt=1-\frac{4}{\pi^2}\left[z-\frac{z^3}{1^2\cdot 3^2\cdot 3}+\frac{z^5}{1^2\cdot 3^2\cdot 5^2\cdot 5}-\dots\right]$$

**12.1.27**

$$\int_0^\infty t^{\mu-\nu-1}\mathbf{H}_\nu(t)dt=\frac{\Gamma(\frac{1}{2}\mu)2^{\mu-\nu-1}\tan(\frac{1}{2}\pi\mu)}{\Gamma(\nu-\frac{1}{2}\mu+1)}$$

$$(|\mathscr{R}\mu|<1,\ \mathscr{R}\nu>\mathscr{R}\mu-\tfrac{3}{2})$$

If $f_\nu(z)=\int_0^z \mathbf{H}_\nu(t)t^\nu dt$

**12.1.28**

$$f_{\nu+1}=(2\nu+1)f_\nu(z)-z^{\nu+1}\mathbf{H}_\nu(z)+\frac{z^{2\nu+2}}{(\nu+1)2^{\nu+1}\Gamma(\frac{1}{2})\Gamma(\nu+\frac{3}{2})}\quad(\mathscr{R}\nu>-\tfrac{1}{2})$$

### Asymptotic Expansions for Large $|z|$

**12.1.29**

$$\mathbf{H}_\nu(z)-Y_\nu(z)=\frac{1}{\pi}\sum_{k=0}^{m-1}\frac{\Gamma(k+\frac{1}{2})}{\Gamma(\nu+\frac{1}{2}-k)\left(\frac{z}{2}\right)^{2k-\nu+1}}+R_m$$

$$(|\arg z|<\pi)$$

where $R_m=O(|z|^{\nu-2m-1})$. If $\nu$ is real, $z$ positive * and $m+\frac{1}{2}-\nu\geq 0$, the remainder after $m$ terms is of the same sign and numerically less than the first term neglected.

**12.1.30**

$$\mathbf{H}_0(z)-Y_0(z)\sim\frac{2}{\pi}\left[\frac{1}{z}-\frac{1}{z^3}+\frac{1^2\cdot 3^2}{z^5}-\frac{1^2\cdot 3^2\cdot 5^2}{z^7}+\dots\right]$$

$$(|\arg z|<\pi)$$

**12.1.31**

$$\mathbf{H}_1(z)-Y_1(z)\sim\frac{2}{\pi}\left[1+\frac{1}{z^2}-\frac{1^2\cdot 3}{z^4}+\frac{1^2\cdot 3^2\cdot 5}{z^6}-\dots\right]$$

$$(|\arg z|<\pi)$$

**12.1.32**

$$\int_0^z[\mathbf{H}_0(t)-Y_0(t)]dt-\frac{2}{\pi}[\ln(2z)+\gamma]$$

$$\sim\frac{2}{\pi}\sum_{k=1}^{\infty}\frac{(-1)^{k+1}(2k)!(2k-1)!}{(k!)^2(2z)^{2k}}\quad(|\arg z|<\pi)$$

where $\gamma=.57721\ 56649\ \dots$ is Euler's constant.

**12.1.33**

$$\int_z^\infty t^{-1}[\mathbf{H}_0(t)-Y_0(t)]dt\sim\frac{2}{\pi z}\sum_{k=0}^{\infty}\frac{(-1)^k[(2k)!]^2}{(k!)^2(2k+1)(2z)^{2k}}$$

$$(|\arg z|<\pi)$$

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*See page II.