11.4.30

$$\int_0^\infty e^{-a^2t^2}Y_{2\nu}(bt)dt=-\frac{\pi^{\frac{1}{2}}}{2a}e^{-\frac{b^2}{8a^2}}\left[I_\nu\left(\frac{b^2}{8a^2}\right)\tan\nu\pi+\frac{1}{\pi}K_\nu\left(\frac{b^2}{8a^2}\right)\sec\nu\pi\right]\qquad\left(|\mathscr{R}\nu|<\frac{1}{2},\mathscr{R}a^2>0\right)$$

11.4.31

$$\int_0^\infty e^{-a^2t^2}I_\nu(bt)dt=\frac{\pi^{\frac{1}{2}}}{2a}e^{\frac{b^2}{8a^2}}I_{\frac{1}{2}\nu}\left(\frac{b^2}{8a^2}\right)\qquad(\mathscr{R}\nu>-1,\mathscr{R}a^2>0)$$

11.4.32

$$\int_0^\infty e^{-a^2t^2}K_0(bt)dt=\frac{\pi^{\frac{1}{2}}}{4a}e^{\frac{b^2}{8a^2}}K_0\left(\frac{b^2}{8a^2}\right)\qquad(\mathscr{R}a^2>0)$$

Weber-Schafheitlin Type Integrals

11.4.33

$$\int_0^\infty\frac{J_\mu(at)J_\nu(bt)dt}{t^\lambda}=\frac{b^\nu\Gamma\left(\frac{\mu+\nu-\lambda+1}{2}\right)}{2^\lambda a^{\nu-\lambda+1}\Gamma(\nu+1)\Gamma\left(\frac{\mu-\nu+\lambda+1}{2}\right)}\times{}_2F_1\left(\frac{\mu+\nu-\lambda+1}{2},\frac{\nu-\mu-\lambda+1}{2};\nu+1;\frac{b^2}{a^2}\right)$$
$$(\mathscr{R}(\mu+\nu-\lambda+1)>0,\mathscr{R}\lambda>-1,0<b<a)$$

11.4.34

$$\int_0^\infty\frac{J_\mu(at)J_\nu(bt)dt}{t^\lambda}=\frac{a^\mu\Gamma\left(\frac{\mu+\nu-\lambda+1}{2}\right)}{2^\lambda b^{\mu-\lambda+1}\Gamma(\mu+1)\Gamma\left(\frac{\nu-\mu+\lambda+1}{2}\right)}\times{}_2F_1\left(\frac{\mu+\nu-\lambda+1}{2},\frac{\mu-\nu-\lambda+1}{2};\mu+1;\frac{a^2}{b^2}\right)$$
$$(\mathscr{R}(\mu+\nu-\lambda+1)>0,\mathscr{R}\lambda>-1,0<a<b)$$

For ${}_2F_1$, see chapter **15.**

Special Cases of the Discontinuous Weber-Schafheitlin Integral

11.4.35

$$\int_0^\infty\frac{J_\mu(at)\sin bt\,dt}{t}=\frac{1}{\mu}\sin\left[\mu\arcsin\frac{b}{a}\right]\qquad(0\le b\le a)$$
$$=\frac{a^\mu\sin\frac{\pi\mu}{2}}{\mu[b+(b^2-a^2)^{\frac{1}{2}}]^\mu}\qquad(b\ge a>0)$$
$$(\mathscr{R}\mu>-1)$$

11.4.36

$$\int_0^\infty\frac{J_\mu(at)\cos bt\,dt}{t}=\frac{1}{\mu}\cos\left[\mu\arcsin\frac{b}{a}\right]\qquad(0\le b\le a)$$
$$=\frac{a^\mu\cos\frac{\pi\mu}{2}}{\mu[b+(b^2-a^2)^{\frac{1}{2}}]^\mu}\qquad(b\ge a>0)$$
$$(\mathscr{R}\mu>0)$$

11.4.37

$$\int_0^\infty J_\mu(at)\cos bt\,dt=\frac{\cos\left[\mu\arcsin\frac{b}{a}\right]}{(a^2-b^2)^{\frac{1}{2}}}\qquad(0\le b<a)$$
$$=\frac{-a^\mu\sin\frac{\pi\mu}{2}}{(b^2-a^2)^{\frac{1}{2}}[b+(b^2-a^2)^{\frac{1}{2}}]^\mu}$$
$$(b>a>0)\qquad(\mathscr{R}\mu>-1)$$

11.4.38

$$\int_0^\infty J_\mu(at)\sin bt\,dt=\frac{\sin\left[\mu\arcsin\frac{b}{a}\right]}{(a^2-b^2)^{\frac{1}{2}}}\qquad(0\le b<a)$$
$$=\frac{a^\mu\cos\frac{\pi\mu}{2}}{(b^2-a^2)^{\frac{1}{2}}[b+(b^2-a^2)^{\frac{1}{2}}]^\mu}$$
$$(b>a>0)\qquad(\mathscr{R}\mu>-2)$$

11.4.39 $$\int_0^\infty e^{ibt}J_0(at)dt=\frac{1}{(a^2-b^2)^{\frac{1}{2}}}\qquad(0\le b<a)$$
$$=\frac{i}{(b^2-a^2)^{\frac{1}{2}}}\qquad(0<a<b)$$

11.4.40

$$\int_0^\infty e^{ibt}Y_0(at)dt=\frac{2i}{\pi(a^2-b^2)^{\frac{1}{2}}}\arcsin\frac{b}{a}\qquad(0\le b<a)$$
$$=\frac{-1}{(b^2-a^2)^{\frac{1}{2}}}+\frac{2i}{\pi(b^2-a^2)^{\frac{1}{2}}}\times\ln\left\{\frac{b-(b^2-a^2)^{\frac{1}{2}}}{a}\right\}\qquad(0<a<b)$$

11.4.41

$$\int_0^\infty t^{\mu-\nu+1}J_\mu(at)J_\nu(bt)dt=0\qquad(0<b<a)$$
$$=\frac{2^{\mu-\nu+1}a^\mu(b^2-a^2)^{\nu-\mu-1}}{b^\nu\Gamma(\nu-\mu)}\qquad(b>a>0)$$
$$(\mathscr{R}\nu>\mathscr{R}\mu>-1)$$

11.4.42 $$\int_0^\infty J_\mu(at)J_{\mu-1}(bt)dt=\frac{b^{\mu-1}}{a^\mu}\qquad(0<b<a)$$
$$=\frac{1}{2b}\qquad(0<b=a)$$
$$=0\qquad(b>a>0)$$
$$(\mathscr{R}\mu>0)$$

11.4.43 $$\int_0^\infty\frac{J_0(at)}{t}\{1-J_0(bt)\}dt=0\qquad(0<b\le a)$$
$$=\ln\frac{b}{a}\qquad(b\ge a>0)$$