Infinite Integrals

Integrals of the Form $\int_0^\infty e^{-pt}t^\mu Z_\nu(t)dt$

11.4.12

$$\int_0^\infty e^{it}t^{\mu-1}J_\nu(t)dt=\frac{e^{\frac{1}{2}i\pi(\mu+\nu)}\,\Gamma(\mu+\nu)\,\Gamma(\frac{1}{2}-\mu)}{\Gamma(\frac{1}{2})2^\mu\Gamma(\nu-\mu+1)}$$

$$\left(\mathscr{R}\mu<\frac{1}{2}, \mathscr{R}(\mu+\nu)>0\right)$$

11.4.13

$$\int_0^\infty e^{-t}t^{\mu-1}I_\nu(t)dt=\frac{\Gamma(\mu+\nu)\Gamma(\frac{1}{2}-\mu)}{\Gamma(\frac{1}{2})2^\mu\Gamma(\nu-\mu+1)}$$

$$\left(\mathscr{R}\mu<\frac{1}{2}, \mathscr{R}(\mu+\nu)>0\right)$$

11.4.14

$$\int_0^\infty \cos bt\, K_0(t)\,dt=\frac{\frac{1}{2}\pi}{(1+b^2)^{\frac{1}{2}}} \qquad (|\mathscr{I}b|<1)$$

11.4.15

$$\int_0^\infty \sin bt\, K_0(t)\,dt=\frac{\text{arc sinh } b}{(1+b^2)^{\frac{1}{2}}} \qquad (|\mathscr{I}b|<1)$$

11.4.16

$$\int_0^\infty t^\mu J_\nu(t)dt=\frac{2^\mu\Gamma\left(\frac{\nu+\mu+1}{2}\right)}{\Gamma\left(\frac{\nu-\mu+1}{2}\right)}$$

$$\left(\mathscr{R}(\mu+\nu)>-1, \mathscr{R}\mu<\frac{1}{2}\right)$$

11.4.17

$$\int_0^\infty J_\nu(t)dt=1 \qquad (\mathscr{R}\nu>-1)$$

11.4.18

$$\int_0^\infty \frac{[1-J_0(t)]dt}{t^\mu}=\frac{\Gamma\left(\frac{\mu-1}{2}\right)\Gamma\left(\frac{3-\mu}{2}\right)}{2^\mu\left\{\Gamma\left(\frac{\mu+1}{2}\right)\right\}^2} \qquad (1<\mathscr{R}\mu<3)$$

11.4.19

$$\int_0^\infty t^\mu Y_\nu(t)dt=\frac{2^\mu}{\pi}\,\Gamma\left(\frac{\mu+\nu+1}{2}\right)\Gamma\left(\frac{\mu-\nu+1}{2}\right)$$

$$\times \sin\frac{\pi}{2}(\mu-\nu)\left(\mathscr{R}(\mu\pm\nu)>-1, \mathscr{R}\mu<\frac{1}{2}\right)$$

11.4.20

$$\int_0^\infty Y_\nu(t)dt=-\tan\frac{\nu\pi}{2} \qquad (|\mathscr{R}\nu|<1)$$

11.4.21

$$\int_0^\infty Y_0(t)dt=0$$

11.4.22

$$\int_0^\infty t^\mu K_\nu(t)dt=2^{\mu-1}\,\Gamma\left(\frac{\mu+\nu+1}{2}\right)\Gamma\left(\frac{\mu-\nu+1}{2}\right)$$

$$(\mathscr{R}(\mu\pm\nu)>-1)$$

11.4.23

$$\int_0^\infty K_0(t)dt=\frac{\pi}{2}$$

11.4.24

$$\int_{-\infty}^\infty e^{-i\omega t}J_n(t)dt=\frac{2(-i)^nT_n(\omega)}{(1-\omega^2)^{\frac{1}{2}}} \quad (\omega^2<1)$$

$$=0(\omega^2>1)$$

where $T_n(\omega)$ is the Chebyshev polynomial of the first kind (see chapter 22).

11.4.25

$$\int_{-\infty}^\infty t^{-1}e^{-i\omega t}J_n(t)dt$$

$$=\frac{2i}{n}(-i)^n(1-\omega^2)^{\frac{1}{2}}U_{n-1}(\omega)\,(\omega^2<1)$$

$$=0(\omega^2>1)$$

where $U_n(\omega)$ is the Chebyshev polynomial of the second kind (see chapter 22).

11.4.26

$$\int_{-\infty}^\infty t^{-\frac{1}{2}}e^{-i\omega t}J_{n+\frac{1}{2}}(t)dt=(-i)^n(2\pi)^{\frac{1}{2}}P_n(\omega)\,(\omega^2<1)$$

$$=0(\omega^2>1)$$

where $P_n(\omega)$ is the Legendre polynomial (see chapter 22).

11.4.27

$$\int_0^\infty e^{-t}t^{\frac{a}{2}-1}J_a[2(zt)^{\frac{1}{2}}]dt=\frac{\gamma(a,z)}{z^{a/2}} \qquad (\mathscr{R}a>0,\ \mathscr{R}z>0)$$

where $\gamma(a, z)$ is the incomplete gamma function (see chapter 6).

Integrals of the Form $\int_0^\infty e^{-a^2t^2}t^\mu Z_\nu(bt)\,dt$

11.4.28

$$\int_0^\infty e^{-a^2t^2}\,t^{\mu-1}J_\nu(bt)dt$$

$$=\frac{\Gamma\left(\frac{1}{2}\nu+\frac{1}{2}\mu\right)\left(\frac{1}{2}\frac{b}{a}\right)^\nu}{2a^\mu\Gamma(\nu+1)}\,M\left(\frac{1}{2}\nu+\frac{1}{2}\mu, \nu+1, -\frac{b^2}{4a^2}\right)$$

$$(\mathscr{R}(\mu+\nu)>0,\ \mathscr{R}a^2>0)$$

where the notation $M(a, b, z)$ stands for the confluent hypergeometric function (see chapter 13).

11.4.29

$$\int_0^\infty e^{-a^2t^2}\,t^{\nu+1}J_\nu(bt)dt$$

$$=\frac{b^\nu}{(2a^2)^{\nu+1}}\,e^{-\frac{b^2}{4a^2}} \qquad (\mathscr{R}\nu>-1,\ \mathscr{R}a^2>0)$$