11.3.13

$$\int_0^z e^{-t}I_n(t)dt=ze^{-z}[I_0(z)+I_1(z)]+n[e^{-z}I_0(z)-1]+2e^{-z}\sum_{k=1}^{n-1}(n-k)I_k(z)$$

11.3.14

$$\int_0^z e^{\pm t}t^{-\nu}I_\nu(t)dt=-\frac{e^{\pm z}z^{-\nu+1}}{2\nu-1}[I_\nu(z)\mp I_{\nu-1}(z)]\mp\frac{1}{2^{\nu-1}(2\nu-1)\Gamma(\nu)}\qquad(\nu\neq\tfrac{1}{2})$$

11.3.15

$$\int_0^z e^{\pm t}t^\nu K_\nu(t)dt=\frac{e^{\pm z}z^{\nu+1}}{2\nu+1}[K_\nu(z)\pm K_{\nu+1}(z)]\mp\frac{2^\nu\Gamma(\nu+1)}{2\nu+1}\qquad(\mathscr{R}\nu>-\tfrac{1}{2})$$

King's integral (see [11.5])

11.3.16 $$\int_0^z e^tK_0(t)dt=ze^z[K_0(z)+K_1(z)]-1$$

11.3.17

$$\int_z^\infty e^tt^{-\nu}K_\nu(t)dt=\frac{e^zz^{-\nu+1}}{2\nu-1}[K_\nu(z)+K_{\nu-1}(z)]\qquad(\mathscr{R}\nu>\tfrac{1}{2})$$

Case 2: $p=0, \mu=\pm\nu$

11.3.18 $$bg_{\nu,\nu-1}(z)=z^\nu Z_\nu(z)$$

11.3.19 $$ag_{-\nu,\nu+1}(z)=-z^{-\nu}Z_\nu(z)$$

11.3.20 $$\int_0^z t^\nu J_{\nu-1}(t)dt=z^\nu J_\nu(z)\qquad(\mathscr{R}\nu>0)$$

11.3.21 $$\int_0^z t^{-\nu}J_{\nu+1}(t)dt=\frac{1}{2^\nu\Gamma(\nu+1)}-z^{-\nu}J_\nu(z)$$

11.3.22

$$2n\int_0^z\frac{J_{2n}(t)dt}{t}=1-\frac{2}{z}\sum_{k=1}^{n}(2k-1)J_{2k-1}(z)=\frac{2}{z}\sum_{k=n+1}^{\infty}(2k-1)J_{2k-1}(z)\qquad(n>0)$$

11.3.23

$$(2n+1)\int_0^z\frac{J_{2n+1}(t)dt}{t}=\int_0^z J_0(t)dt-J_1(z)-\frac{4}{z}\sum_{k=1}^{n}kJ_{2k}(z)$$

11.3.24

$$\int_0^z t^\nu Y_{\nu-1}(t)dt=z^\nu Y_\nu(z)+\frac{2^\nu\Gamma(\nu)}{\pi}\qquad(\mathscr{R}\nu>0)$$

11.3.25 $$\int_0^z t^\nu I_{\nu-1}(t)dt=z^\nu I_\nu(z)\qquad(\mathscr{R}\nu>0)$$

11.3.26 $$\int_0^z t^{-\nu}I_{\nu+1}(t)dt=z^{-\nu}I_\nu(z)-\frac{1}{2^\nu\Gamma(\nu+1)}$$

11.3.27

$$\int_0^z t^\nu K_{\nu-1}(t)dt=-z^\nu K_\nu(z)+2^{\nu-1}\Gamma(\nu)\qquad(\mathscr{R}\nu>0)$$

11.3.28 $$\int_z^\infty t^{-\nu}K_{\nu+1}(t)dt=z^{-\nu}K_\nu(z)$$

Indefinite Integrals of Products of Bessel Functions

Let $\mathscr{C}_\mu(z)$ and $\mathscr{D}_\nu(z)$ denote any two cylinder functions of orders μ and ν respectively.

11.3.29

$$\int^z\left\{(k^2-l^2)t-\frac{(\mu^2-\nu^2)}{t}\right\}\mathscr{C}_\mu(kt)\mathscr{D}_\nu(lt)dt=z\{k\mathscr{C}_{\mu+1}(kz)\mathscr{D}_\nu(lz)-l\mathscr{C}_\mu(kz)\mathscr{D}_{\nu+1}(lz)\}-(\mu-\nu)\mathscr{C}_\mu(kz)\mathscr{D}_\nu(lz)$$ *

11.3.30

$$\int^z t^{-\mu-\nu-1}\mathscr{C}_{\mu+1}(t)\mathscr{D}_{\nu+1}(t)dt=-\frac{z^{-\mu-\nu}}{2(\mu+\nu+1)}\{\mathscr{C}_\mu(z)\mathscr{D}_\nu(z)+\mathscr{C}_{\mu+1}(z)\mathscr{D}_{\nu+1}(z)\}$$

11.3.31

$$\int^z t^{\mu+\nu+1}\mathscr{C}_\mu(t)\mathscr{D}_\nu(t)dt=\frac{z^{\mu+\nu+2}}{2(\mu+\nu+1)}\{\mathscr{C}_\mu(z)\mathscr{D}_\nu(z)+\mathscr{C}_{\mu+1}(z)\mathscr{D}_{\nu+1}(z)\}$$

11.3.32

$$\int_0^z tJ_{\nu-1}^2(t)dt=2\sum_{k=0}^{\infty}(\nu+2k)J_{\nu+2k}^2(z)\qquad(\mathscr{R}\nu>0)$$

11.3.33

$$\int_0^z t[J_{\nu-1}^2(t)-J_{\nu+1}^2(t)]dt=2\nu J_\nu^2(z)\qquad(\mathscr{R}\nu>0)$$

11.3.34 $$\int_0^z tJ_0^2(t)dt=\frac{z^2}{2}[J_0^2(z)+J_1^2(z)]$$

11.3.35

$$\int_0^z J_n(t)J_{n+1}(t)dt=\frac{1}{2}[1-J_0^2(z)]-\sum_{k=1}^{n}J_k^2(z)=\sum_{k=n+1}^{\infty}J_k^2(z)$$

*See page II.