11.1.14 $x^{\frac{1}{2}}e^{-x}\int_0^x I_0(t)dt \sim (2\pi)^{-\frac{1}{2}}\sum_{k=0}^{\infty} a_k x^{-k}$

where the a_k are defined as in **11.1.12.**

11.1.15 $x^{\frac{1}{2}}e^{x}\int_x^\infty K_0(t)dt \sim \left(\frac{\pi}{2}\right)^{\frac{1}{2}}\sum_{k=0}^{\infty} (-)^k a_k x^{-k}$

where the a_k are defined as in **11.1.12.**

Polynomial Approximations [2]

11.1.16 $8 \le x \le \infty$

$$\int_x^\infty [J_0(t)+iY_0(t)]dt$$

$$=x^{-\frac{1}{2}}e^{i(x-\pi/4)}\left[\sum_{k=0}^{7} (-)^k a_k (x/8)^{-2k-1} + i\sum_{k=0}^{7} (-)^k b_k (x/8)^{-2k} + \epsilon(x)\right]$$

$$|\epsilon(x)| \le 2\times 10^{-9}$$

k	a_k	b_k
0	.06233 47304	.79788 45600
1	.00404 03539	.01256 42405
2	.00100 89872	.00178 70944
3	.00053 66169	.00067 40148
4	.00039 92825	.00041 00676
5	.00027 55037	.00025 43955
6	.00012 70039	.00011 07299
7	.00002 68482	.00002 26238

11.1.17 $8 \le x \le \infty$

$$x^{\frac{1}{2}}e^{-x}\int_0^x I_0(t)\,dt = \sum_{k=0}^{6} d_k (x/8)^{-k} + \epsilon(x)$$

$$|\epsilon(x)| \le 2\times 10^{-6}$$

k	d_k
0	.39894 23
1	.03117 34
2	.00591 91
3	.00559 56
4	−.01148 58
5	.01774 40
6	−.00739 95

[2] Approximation **11.1.16** is from A. J. M. Hitchcock. Polynomial approximations to Bessel functions of order zero and one and to related functions, Math. Tables Aids Comp. 11, 86–88 (1957) (with permission).

11.1.18 $7 \le x \le \infty$

$$x^{\frac{1}{2}}e^{x}\int_x^\infty K_0(t)dt = \sum_{k=0}^{6} (-)^k e_k (x/7)^{-k} + \epsilon(x)$$

$$|\epsilon(x)| \le 2\times 10^{-7}$$

k	e_k
0	1.25331 414
1	0.11190 289
2	.02576 646
3	.00933 994
4	.00417 454
5	.00163 271
6	.00033 934

$\int \frac{J_0(t)dt}{t}, \int \frac{Y_0(t)dt}{t}, \int \frac{K_0(t)dt}{t}$

11.1.19

$$\int_0^x \frac{1-J_0(t)}{t}\,dt$$

$$=2x^{-1}\sum_{k=0}^{\infty} (2k+3)[\psi(k+2)-\psi(1)]\,J_{2k+3}(x)$$

$$=1-2x^{-1}J_1(x) + 2x^{-1}\sum_{k=0}^{\infty} (2k+5)[\psi(k+3)-\psi(1)-1]J_{2k+5}(x)$$

For $\psi(z)$, see **6.3.**

11.1.20

$$\int_x^\infty \frac{J_0(t)dt}{t}+\gamma+\ln\frac{x}{2}=\int_0^x \frac{[1-J_0(t)]dt}{t} = -\sum_{k=1}^{\infty}\frac{(-)^k\left(\frac{x}{2}\right)^{2k}}{2k(k!)^2}$$

11.1.21

$$\int_x^\infty \frac{Y_0(t)dt}{t} = -\frac{1}{\pi}\left(\ln\frac{x}{2}\right)^2 - \frac{2\gamma}{\pi}\left(\ln\frac{x}{2}\right)+\frac{1}{\pi}\left(\frac{\pi^2}{6}-\gamma^2\right) + \frac{2}{\pi}\sum_{k=1}^{\infty}\frac{(-)^k\left(\frac{x}{2}\right)^{2k}}{2k(k!)^2}\left\{\psi(k+1)+\frac{1}{2k}-\ln\frac{x}{2}\right\}$$

11.1.22

$$\int_x^\infty \frac{K_0(t)dt}{t} = \frac{1}{2}\left(\ln\frac{x}{2}\right)^2+\gamma\ln\frac{x}{2}+\frac{\pi^2}{24}+\frac{\gamma^2}{2} - \sum_{k=1}^{\infty}\frac{\left(\frac{x}{2}\right)^{2k}}{2k(k!)^2}\left\{\psi(k+1)+\frac{1}{2k}-\ln\frac{x}{2}\right\}$$

11.1.23

$$\int_{-ix}^{-i\infty}\frac{K_0(t)dt}{t} = \frac{i\pi}{2}\int_x^\infty \frac{J_0(t)dt}{t} - \frac{\pi}{2}\int_x^\infty \frac{Y_0(t)dt}{t}$$