The recurrence relation **10.2.18** yields successively

$$-\sqrt{\tfrac{1}{2}\pi/3.6}K_{5/2}(3.6)=-.01192\ 222-\frac{3}{3.6}(.01523\ 3952)=-.02461\ 718$$

$$\sqrt{\tfrac{1}{2}\pi/3.6}K_{7/2}(3.6)=.01523\ 3952+\frac{5}{3.6}(.02461\ 718)=.04942\ 4480$$

$$-\sqrt{\tfrac{1}{2}\pi/3.6}K_{9/2}(3.6)=-.02461\ 718-\frac{7}{3.6}(.04942\ 4480)=-.12072\ 034$$

$$\sqrt{\tfrac{1}{2}\pi/3.6}K_{11/2}(3.6)=.04942\ 4480+\frac{9}{3.6}(.12072\ 034)=.35122\ 533.$$

As a check, the recurrence can be carried out until $n=9$ and the value of $\sqrt{\tfrac{1}{2}\pi/3.6}K_{19/2}(3.6)$ so obtained can be compared with the corresponding value from **Table 10.9.**

To compute $\sqrt{\tfrac{1}{2}\pi/x}I_{n+\frac{1}{2}}(x)$ when both n and x are outside the range of **Table 10.9,** use the device described in [9.20].

Airy Functions

To compute Ai(x), Bi(x) for values of x beyond 1, use auxiliary functions from **Table 10.11.**

Example 5. Compute Ai(x) for $x=4.5$.

First, for $x=4.5$,

$$\xi=\tfrac{2}{3}x^{3/2}=6.36396\ 1029,\ \xi^{-1}=.15713\ 48403.$$

Hence, from **Table 10.11,** $f(-\xi)=.55848\ 24$ and thus

$$\begin{aligned}\mathrm{Ai}(4.5)&=\tfrac{1}{2}(4.5)^{-1/4}(.55848\ 24)\exp(-6.36396\ 1029)\\&=\tfrac{1}{2}(.68658\ 905)(.55848\ 24)(.00172\ 25302)\\&=.00033\ 02503.\end{aligned}$$

To compute the zeros c, c' of a solution $y(x)$ of the equation $y''-xy=0$ and of its derivative $y'(x)$, respectively, the following formulas may be used, in which d, d' denote approximations to c, c' and $u=y(d)/y'(d)$, $v=y'(d')/d'^2y(d')$.

$$c=d-u-2d\frac{u^3}{3!}+2\frac{u^4}{4!}-24d^2\frac{u^5}{5!}+88d\frac{u^6}{6!}-(88+720d^3)\frac{u^7}{7!}+5856d^2\frac{u^8}{8!}-(16640d+40320d^4)\frac{u^9}{9!}+\ldots$$

$$c'=d'\left\{1-v-\frac{v^2}{2!}-(3+2d'^3)\frac{v^3}{3!}-(15+10d'^3)\frac{v^4}{4!}-(105+76d'^3+24d'^6)\frac{v^5}{5!}-(945+756d'^3+272d'^6)\frac{v^6}{6!}-\ldots\right\}$$

$$y'(c)=y'(d)\left\{1-d\frac{u^2}{2!}+\frac{u^3}{3!}-3d^2\frac{u^4}{4!}+14d\frac{u^5}{5!}-(14+45d^3)\frac{u^6}{6!}+471d^2\frac{u^7}{7!}-(1432d+1575d^4)\frac{u^8}{8!}+\ldots\right\}$$

$$y(c')=y(d')\left\{1-d'^2\frac{v^2}{2!}-d'^3\frac{v^3}{3!}-(3d'^3+3d'^6)\frac{v^4}{4!}-(15d'^3+14d'^6)\frac{v^5}{5!}-(105d'^3+101d'^6+45d'^4)\frac{v^6}{6!}-\ldots\right\}$$

Example 6. Compute the zero of $y(x)=\mathrm{Ai}(x)-\mathrm{Bi}(x)$ near $d=-.4$.

From **Table 10.11,**

$$y(-.4)=.02420\ 467,\ y'(-.4)=-.71276\ 627$$

whence $u=y(-.4)/y'(-.4)=-.03395\ 8776$. From the above formulas

$$\begin{aligned}c&=-.4+.03395\ 8776-.00000\ 5221+.00000\ 0111+.00000\ 0001\\&=-.36604\ 6333.\end{aligned}$$

$$\begin{aligned}y'(c)&=(-.71276\ 627)\{1+.00023\ 0640-.00000\ 6527-.00000\ 0027+.00000\ 0002\}\\&=(-.71276\ 627)(1.00022\ 4088)\\&=-.71292\ 599.\end{aligned}$$