10.4.62

$$\mathrm{Ai}'(-z)\sim-\pi^{-\frac{1}{2}}z^{\frac{1}{4}}\left[\cos\left(\zeta+\frac{\pi}{4}\right)\sum_0^\infty(-1)^k d_{2k}\zeta^{-2k}+\sin\left(\zeta+\frac{\pi}{4}\right)\sum_0^\infty(-1)^k d_{2k+1}\zeta^{-2k-1}\right] \quad (|\arg z|<\tfrac{2}{3}\pi)$$

10.4.63

$$\mathrm{Bi}(z)\sim\pi^{-\frac{1}{2}}z^{-\frac{1}{4}}e^{\zeta}\sum_0^\infty c_k\zeta^{-k} \qquad (|\arg z|<\tfrac{1}{3}\pi)$$

10.4.64

$$\mathrm{Bi}(-z)\sim\pi^{-\frac{1}{2}}z^{-\frac{1}{4}}\left[\cos\left(\zeta+\frac{\pi}{4}\right)\sum_0^\infty(-1)^k c_{2k}\zeta^{-2k}+\sin\left(\zeta+\frac{\pi}{4}\right)\sum_0^\infty(-1)^k c_{2k+1}\zeta^{-2k-1}\right] \quad (|\arg z|<\tfrac{2}{3}\pi)$$

10.4.65

$$\mathrm{Bi}(ze^{\pm\pi i/3})\sim\sqrt{2/\pi}e^{\pm\pi i/6}z^{-\frac{1}{4}}\left[\sin\left(\zeta+\frac{\pi}{4}\mp\frac{i}{2}\ln 2\right)\sum_0^\infty(-1)^k c_{2k}\zeta^{-2k}-\cos\left(\zeta+\frac{\pi}{4}\mp\frac{i}{2}\ln 2\right)\sum_0^\infty(-1)^k c_{2k+1}\zeta^{-2k-1}\right] \quad (|\arg z|<\tfrac{2}{3}\pi)$$

10.4.66

* $$\mathrm{Bi}'(z)\sim\pi^{-\frac{1}{2}}z^{\frac{1}{4}}e^{\zeta}\sum_0^\infty d_k\zeta^{-k} \quad (|\arg z|<\tfrac{1}{3}\pi)$$

10.4.67

$$\mathrm{Bi}'(-z)\sim\pi^{-\frac{1}{2}}z^{\frac{1}{4}}\left[\sin\left(\zeta+\frac{\pi}{4}\right)\sum_0^\infty(-1)^k d_{2k}\zeta^{-2k}-\cos\left(\zeta+\frac{\pi}{4}\right)\sum_0^\infty(-1)^k d_{2k+1}\zeta^{-2k-1}\right] \quad (|\arg z|<\tfrac{2}{3}\pi)$$

10.4.68

$$\mathrm{Bi}'(ze^{\pm\pi i/3})\sim\sqrt{2/\pi}e^{\mp\pi i/6}z^{\frac{1}{4}}\left[\cos\left(\zeta+\frac{\pi}{4}\mp\frac{i}{2}\ln 2\right)\sum_0^\infty(-1)^k d_{2k}\zeta^{-2k}+\sin\left(\zeta+\frac{\pi}{4}\mp\frac{i}{2}\ln 2\right)\sum_0^\infty(-1)^k d_{2k+1}\zeta^{-2k-1}\right] \quad (|\arg z|<\tfrac{2}{3}\pi)$$

Modulus and Phase

10.4.69

$$\mathrm{Ai}(-x)=M(x)\cos\theta(x),\ \mathrm{Bi}(-x)=M(x)\sin\theta(x)$$
$$M(x)=\sqrt{[\mathrm{Ai}^2(-x)+\mathrm{Bi}^2(-x)]},\quad \theta(x)=\arctan[\mathrm{Bi}(-x)/\mathrm{Ai}(-x)]$$

10.4.70

$$\mathrm{Ai}'(-x)=N(x)\cos\phi(x),\ \mathrm{Bi}'(-x)=N(x)\sin\phi(x)$$
$$N(x)=\sqrt{[\mathrm{Ai}'^2(-x)+\mathrm{Bi}'^2(-x)]},\quad \phi(x)=\arctan[\mathrm{Bi}'(-x)/\mathrm{Ai}'(-x)]$$

Differential Equations for Modulus and Phase

Primes denote differentiation with respect to x

10.4.71 $$M^2\theta'=-\pi^{-1},\ N^2\phi'=-\pi^{-1}x$$

10.4.72 $$N^2=M'^2+M^2\theta'^2=M'^2+\pi^{-2}M^{-2}$$ *

10.4.73 $$NN'=-xMM'$$

10.4.74

$$\tan(\phi-\theta)=M\theta'/M'=-(\pi MM')^{-1},\quad MN\sin(\phi-\theta)=\pi^{-1}$$

10.4.75 $$M''+xM-\pi^{-2}M^{-3}=0$$

10.4.76 $$(M^2)'''+4x(M^2)'-2M^2=0$$

10.4.77 $$\theta'^2+\tfrac{1}{2}(\theta'''/\theta')-\tfrac{3}{4}(\theta''/\theta')^2=x$$

Asymptotic Expansions of Modulus and Phase for Large x

10.4.78 $$M^2(x)\sim\frac{1}{\pi}x^{-1/2}\sum_0^\infty\frac{(-1)^k}{12^k k!}2^{3k}\left(\frac{1}{2}\right)_{3k}(2x)^{-3k}$$

10.4.79

$$\theta(x)\sim\frac{1}{4}\pi-\frac{2}{3}x^{3/2}\left[1-\frac{5}{4}(2x)^{-3}+\frac{1105}{96}(2x)^{-6}-\frac{82825}{128}(2x)^{-9}+\frac{12820\,31525}{14336}(2x)^{-12}-\ldots\right]$$

10.4.80

$$N^2(x)\sim\frac{1}{\pi}x^{\frac{1}{2}}\sum_0^\infty\frac{(-1)^{k+1}}{12^k k!}\frac{6k+1}{6k-1}2^{3k}\left(\frac{1}{2}\right)_{3k}(2x)^{-3k}$$

10.4.81

$$\phi(x)\sim\frac{3}{4}\pi-\frac{2}{3}x^{3/2}\left[1+\frac{7}{4}(2x)^{-3}-\frac{1463}{96}(2x)^{-6}+\frac{495271}{640}(2x)^{-9}-\frac{2065\,30429}{2048}(2x)^{-12}+\ldots\right]$$

Asymptotic Forms of $\int_0^x\mathrm{Ai}(\pm t)\,dt$, $\int_0^x\mathrm{Bi}(\pm t)\,dt$ for Large x

10.4.82 $$\int_0^x\mathrm{Ai}(t)\,dt\sim\frac{1}{3}-\frac{1}{2}\pi^{-1/2}x^{-3/4}\exp\left(-\frac{2}{3}x^{3/2}\right)$$

10.4.83

$$\int_0^x\mathrm{Ai}(-t)\,dt\sim\frac{2}{3}-\pi^{-1/2}x^{-3/4}\cos\left(\frac{2}{3}x^{3/2}+\frac{\pi}{4}\right)$$

*See page II.