

Modified Spherical Bessel Functions of the Third Kind

10.2.15

$$\begin{aligned} \sqrt{\frac{1}{2}\pi/z}K_{n+\frac{1}{2}}(z) &= \frac{1}{2}\pi i e^{(n+1)\pi i/2} h_n^{(1)}(ze^{\frac{1}{2}\pi i}) & (-\pi < \arg z \leq \frac{1}{2}\pi) \\ &= -\frac{1}{2}\pi i e^{-(n+1)\pi i/2} h_n^{(2)}(ze^{-\frac{1}{2}\pi i}) & (\frac{1}{2}\pi < \arg z \leq \pi) \\ &= (\frac{1}{2}\pi/z)e^{-z} \sum_0^n (n+\frac{1}{2}, k)(2z)^{-k} \end{aligned}$$

10.2.16

$$K_{n+\frac{1}{2}}(z) = K_{-n-\frac{1}{2}}(z) \quad (n=0, 1, 2, \dots)$$

The Functions $\sqrt{\frac{1}{2}\pi/z}K_{n+\frac{1}{2}}(z), n=0, 1, 2$

$$\begin{aligned} 10.2.17 \quad \sqrt{\frac{1}{2}\pi/z}K_{1/2}(z) &= (\frac{1}{2}\pi/z)e^{-z} \\ \sqrt{\frac{1}{2}\pi/z}K_{3/2}(z) &= (\frac{1}{2}\pi/z)e^{-z}(1+z^{-1}) \\ \sqrt{\frac{1}{2}\pi/z}K_{5/2}(z) &= (\frac{1}{2}\pi/z)e^{-z}(1+3z^{-1}+3z^{-2}) \end{aligned}$$

Elementary Properties

Recurrence Relations

$$f_n(z): \sqrt{\frac{1}{2}\pi/z}I_{n+\frac{1}{2}}(z), (-1)^{n+1}\sqrt{\frac{1}{2}\pi/z}K_{n+\frac{1}{2}}(z) \quad (n=0, \pm 1, \pm 2, \dots)$$

$$10.2.18 \quad f_{n-1}(z) - f_{n+1}(z) = (2n+1)z^{-1}f_n(z)$$

$$10.2.19 \quad nf_{n-1}(z) + (n+1)f_{n+1}(z) = (2n+1)\frac{d}{dz}f_n(z)$$

$$10.2.20 \quad \frac{n+1}{z}f_n(z) + \frac{d}{dz}f_n(z) = f_{n-1}(z)$$

(See 10.2.22.)

$$10.2.21 \quad -\frac{n}{z}f_n(z) + \frac{d}{dz}f_n(z) = f_{n+1}(z)$$

(See 10.2.23.)

Differentiation Formulas

$$f_n(z): \sqrt{\frac{1}{2}\pi/z}I_{n+\frac{1}{2}}(z), (-1)^{n+1}\sqrt{\frac{1}{2}\pi/z}K_{n+\frac{1}{2}}(z) \quad (n=0, \pm 1, \pm 2, \dots)$$

$$10.2.22 \quad \left(\frac{1}{z}\frac{d}{dz}\right)^m [z^{n+\frac{1}{2}}f_n(z)] = z^{n-m+\frac{1}{2}}f_{n-m}(z)$$

$$10.2.23 \quad \left(\frac{1}{z}\frac{d}{dz}\right)^m [z^{-n-\frac{1}{2}}f_n(z)] = z^{-n-m-\frac{1}{2}}f_{n+m}(z) \quad (m=1, 2, 3, \dots)$$

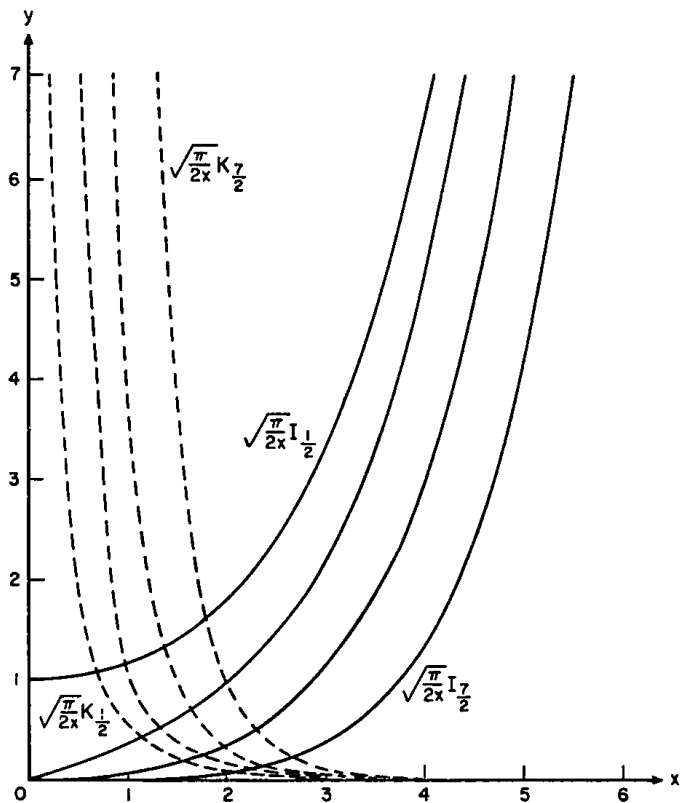


FIGURE 10.4. $\sqrt{\frac{\pi}{2x}}I_{n+\frac{1}{2}}(x), \sqrt{\frac{\pi}{2x}}K_{n+\frac{1}{2}}(x), n=0(1)3.$

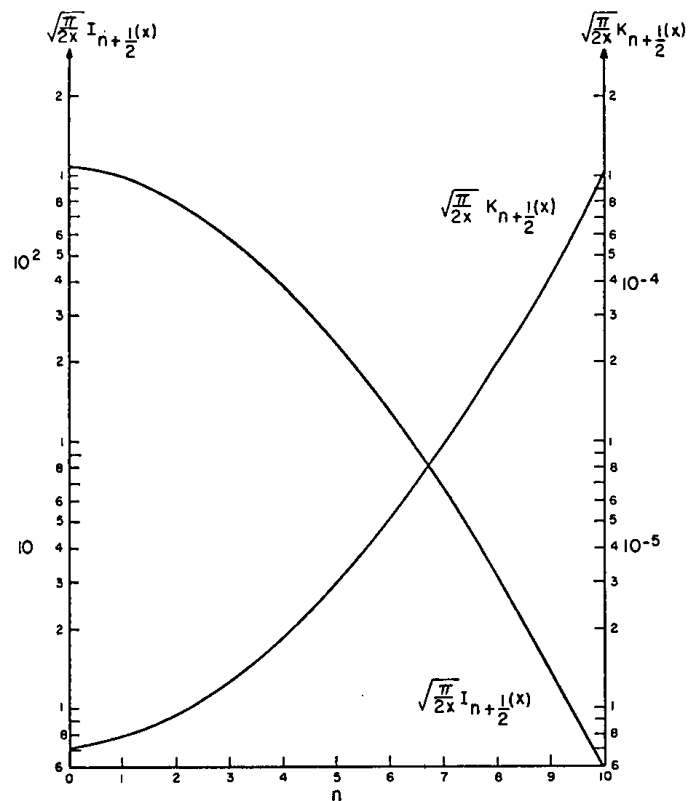


FIGURE 10.5. $\sqrt{\frac{\pi}{2x}}I_{n+\frac{1}{2}}(x), \sqrt{\frac{\pi}{2x}}K_{n+\frac{1}{2}}(x), x=10.$