McMahon's Expansions for n Fixed and s Large

10.1.58

$$a'_{n,s},\ b'_{n,s}\sim\beta-(\mu+7)(8\beta)^{-1}$$
$$-\frac{4}{3}(7\mu^2+154\mu+95)(8\beta)^{-3}$$
$$-\frac{32}{15}(85\mu^3+3535\mu^2+3561\mu+6133)(8\beta)^{-5}$$
$$-\frac{64}{105}(6949\mu^4+474908\mu^3+330638\mu^2$$
$$+9046780\mu-5075147)(8\beta)^{-7}-\ldots$$

$\beta=\pi(s+\frac{1}{2}n-\frac{1}{2})$ for $a'_{n,s}$, $\beta=\pi(s+\frac{1}{2}n)$ for $b'_{n,s}$;
$\mu=(2n+1)^2$

Asymptotic Expansions of Zeros and Associated Values for n Large

10.1.59

$$a'_{n,1}\sim(n+\tfrac{1}{2})+.8086165(n+\tfrac{1}{2})^{1/3}-.236680(n+\tfrac{1}{2})^{-1/3}$$
$$-.20736(n+\tfrac{1}{2})^{-1}+.0233(n+\tfrac{1}{2})^{-5/3}+\ldots$$

10.1.60

$$b'_{n,1}\sim(n+\tfrac{1}{2})+1.8210980(n+\tfrac{1}{2})^{1/3}$$
$$+.802728(n+\tfrac{1}{2})^{-1/3}-.11740(n+\tfrac{1}{2})^{-1}$$
$$+.0249(n+\tfrac{1}{2})^{-5/3}+\ldots$$

10.1.61

$$j_n(a'_{n,1})\sim.8458430(n+\tfrac{1}{2})^{-5/6}\{1-.566032(n+\tfrac{1}{2})^{-2/3}$$
$$+.38081(n+\tfrac{1}{2})^{-4/3}-.2203(n+\tfrac{1}{2})^{-2}+\ldots\}$$

10.1.62

$$y_n(b'_{n,1})\sim.7183921(n+\tfrac{1}{2})^{-5/6}\{1-1.274769(n+\tfrac{1}{2})^{-2/3}$$
$$+1.23038(n+\tfrac{1}{2})^{-4/3}-1.0070(n+\tfrac{1}{2})^{-2}+\ldots\}$$

See [10.31] for corresponding expansions for $s=2, 3$.

Uniform Asymptotic Expansions of Zeros and Associated Values for n Large

10.1.63

$$a'_{n,s}\sim(n+\tfrac{1}{2})\{z[(n+\tfrac{1}{2})^{-2/3}a'_s]$$
$$+\sum_{k=1}^{\infty}h_k[(n+\tfrac{1}{2})^{-2/3}a'_s](n+\tfrac{1}{2})^{-2k}\}$$

10.1.64

$$b'_{n,s}\sim(n+\tfrac{1}{2})\{z[(n+\tfrac{1}{2})^{-2/3}b'_s]$$
$$+\sum_{k=1}^{\infty}h_k[(n+\tfrac{1}{2})^{-2/3}b'_s](n+\tfrac{1}{2})^{-2k}\}$$

10.1.65

$$j_n(a'_{n,s})\sim\sqrt{\tfrac{1}{2}\pi}\,\mathrm{Ai}(a'_s)(n+\tfrac{1}{2})^{-5/6}$$
$$h[(n+\tfrac{1}{2})^{-2/3}a'_s]\,(z[(n+\tfrac{1}{2})^{-2/3}a'_s])^{-1/2}$$
$$\{1+\sum_{k=1}^{\infty}H_k[(n+\tfrac{1}{2})^{-2/3}a'_s](n+\tfrac{1}{2})^{-2k}\}$$

10.1.66

$$y_n(b'_{n,s})\sim-\sqrt{\tfrac{1}{2}\pi}\,\mathrm{Bi}(b'_s)(n+\tfrac{1}{2})^{-5/6}$$
$$h[(n+\tfrac{1}{2})^{-2/3}b'_s]\,(z[(n+\tfrac{1}{2})^{-2/3}b'_s])^{-1/2}$$
$$\{1+\sum_{k=1}^{\infty}H_k[(n+\tfrac{1}{2})^{-2/3}b'_s](n+\tfrac{1}{2})^{-2k}\}$$

$h(\xi)$, $z(\xi)$ are defined as in **9.5.26, 9.3.38, 9.3.39.**
a'_s, b'_s s-th (negative) real zero of $\mathrm{Ai}'(z)$, $\mathrm{Bi}'(z)$ (see **10.4.95, 10.4.99.**)

Complex Zeros of $h_n^{(1)}(z)$, $h_n^{(1)\prime}(z)$

$h_n^{(1)}(z)$ and $h_n^{(1)}(ze^{2m\pi i})$, m any integer, have the same zeros.

$h_n^{(1)}(z)$ has n zeros, symmetrically distributed with respect to the imaginary axis and lying approximately on the finite arc joining $z=-n$ and $z=n$ shown in **Figure 9.6.** If n is odd, one zero lies on the imaginary axis.

$h_n^{(1)\prime}(z)$ has $n+1$ zeros lying approximately on the same curve. If n is even, one zero lies on the imaginary axis.