Hence

$$\zeta=-.5567724 \text{ and } \left(\frac{4\zeta}{1-z^2}\right)^{1/4}=+1.155332.$$

Next,

$$\nu^{1/3}=3.684031, \qquad \nu^{2/3}\zeta=-7.556562.$$

Interpolating in **Table 10.11**, we find that

$$\mathrm{Ai}(\nu^{2/3}\zeta)=+.299953, \qquad \mathrm{Ai}'(\nu^{2/3}\zeta)=+.451441,$$

$$\mathrm{Bi}(\nu^{2/3}\zeta)=-.160565, \qquad \mathrm{Bi}'(\nu^{2/3}\zeta)=+.819542.$$

As a check on the interpolation, we may verify that $\mathrm{Ai}\,\mathrm{Bi}'-\mathrm{Ai}'\mathrm{Bi}=1/\pi$.

Interpolating in the table following **9.3.46** we obtain

$$b_0(\zeta)=+.0136, \qquad c_0(\zeta)=+.1442.$$

The contributions of the terms involving $a_1(\zeta)$ and $d_1(\zeta)$ are negligible, and substituting in the asymptotic expansions we find that

$$J_{50}(75)=+1.155332(50^{-1/3}\times.299953 +50^{-5/3}\times.451441\times.0136)=+.094077,$$

$$J'_{50}(75)=-(4/3)(1.155332)^{-1}(50^{-4/3}\times.299953 \times.1442+50^{-2/3}\times.451441)=-.038658,$$

$$Y_{50}(75)=-1.155332(-50^{-1/3}\times.160565 +50^{-5/3}\times.819542\times.0136)=+.050335,$$

$$Y'_{50}(75)=+(4/3)(1.155332)^{-1}(-50^{-4/3}\times.160565 \times.1442+50^{-2/3}\times.819542)=+.069543.$$

As a check we may verify that

$$JY'-J'Y=2/(75\pi).$$

Remarks. This example may also be computed using the Debye expansions **9.3.15, 9.3.16, 9.3.19,** and **9.3.20.** Four terms of each of these series are required, compared with two in the computations above. The closer the argument-order ratio is to unity, the less effective the Debye expansions become. In the neighborhood of unity the expansions **9.3.23, 9.3.24, 9.3.27,** and **9.3.28** will furnish results of moderate accuracy; for high-accuracy work the uniform expansions should again be used.

Example 5. To evaluate the 5th positive zero of $J_{10}(x)$ and the corresponding value of $J'_{10}(x)$, each to 5 decimals.

We use the asymptotic expansions **9.5.22** and **9.5.23** setting $\nu=10$, $s=5$. From **Table 10.11** we find

$$a_5=-7.944134, \qquad \mathrm{Ai}'(a_5)=+.947336.$$

Hence

$$\zeta=10^{-2/3}a_5=.21544347a_5=-1.7115118.$$

Interpolating in the table following **9.5.26** we obtain

$$z(\zeta)=+2.888631, \qquad h(\zeta)=+.98259,$$
$$f_1(\zeta)=+.0107, \qquad F_1(\zeta)=-.001.$$

The bounds given at the foot of the table show that the contributions of higher terms to the asymptotic series are negligible. Hence

$$j_{10,5}=28.88631+.00107+\ .\ .\ .\ =28.88738,$$

$$J'_{10}(j_{10,5})=-\frac{2}{10^{2/3}}\frac{.947336}{2.888631\times.98259} \times(1-.00001+\ .\ .\ .)=-.14381.$$

Example 6. To evaluate the first root of $J_0(x)Y_0(\lambda x)-Y_0(x)J_0(\lambda x)=0$ for $\lambda=\frac{3}{2}$ to 4 significant figures.

Let $\alpha_\lambda^{(1)}$ denote the root. Direct interpolation in **Table 9.7** is impracticable owing to the divergence of the differences. Inspection of **9.5.28** suggests that a smoother function is $(\lambda-1)\alpha_\lambda^{(1)}$. Using **Table 9.7** we compute the following values

$1/\lambda$	$(\lambda-1)\alpha_\lambda^{(1)}$	δ	δ^2
0.4	3.110		
		+21	
0.6	3.131		−12
		+9	
0.8	3.140		−7
		+2	
1.0	3.142(π)		

Interpolating for $1/\lambda=.667$, we obtain $(\lambda-1)\alpha_\lambda^{(1)}=3.134$ and thence the required root $\alpha_{1.5}^{(1)}=6.268$.

Example 7. To evaluate $\mathrm{ber}_n\,1.55$, $\mathrm{bei}_n\,1.55$, $n=0, 1, 2, \ldots$, each to 5 decimals.

We use the recurrence relation

$$J_{n-1}(xe^{3\pi i/4})+J_{n+1}(xe^{3\pi i/4}) =-\frac{n\sqrt{2}}{x}(1+i)J_n(xe^{3\pi i/4}),$$

taking arbitrary values zero for $J_9(xe^{3\pi i/4})$ and $1+0i$ for $J_8(xe^{3\pi i/4})$ (see **Example 1**).