9.10.7

$$g_\nu(\pm x)$$
$$\sim\sum_{k=1}^{\infty}(\mp)^k\frac{(\mu-1)(\mu-9)\ldots\{\mu-(2k-1)^2\}}{k!(8x)^k}\sin\left(\frac{k\pi}{4}\right)$$

The terms[6] in $\text{ker}_\nu x$ and $\text{kei}_\nu x$ in equations **9.10.1** and **9.10.2** are asymptotically negligible compared with the other terms, but their inclusion in numerical calculations yields improved accuracy.

The corresponding series for $\text{ber}'_\nu x$, $\text{bei}'_\nu x$, $\text{ker}'_\nu x$ and $\text{kei}'_\nu x$ can be derived from **9.2.11** and **9.2.13** with $z=xe^{3\pi i/4}$; the extra terms in the expansions of $\text{ber}'_\nu x$ and $\text{bei}'_\nu x$ are respectively

$$-(1/\pi)\{\sin(2\nu\pi)\text{ker}'_\nu x+\cos(2\nu\pi)\text{kei}'_\nu x\}$$

and

$$(1/\pi)\{\cos(2\nu\pi)\text{ker}'_\nu x-\sin(2\nu\pi)\text{kei}'_\nu x\}.$$

Modulus and Phase

9.10.8

$$M_\nu=\sqrt{(\text{ber}_\nu^2 x+\text{bei}_\nu^2 x)},\qquad \theta_\nu=\arctan\,(\text{bei}_\nu x/\text{ber}_\nu x)$$

9.10.9 $\quad \text{ber}_\nu x=M_\nu\cos\theta_\nu,\qquad \text{bei}_\nu x=M_\nu\sin\theta_\nu$

9.10.10 $\quad M_{-n}=M_n,\qquad \theta_{-n}=\theta_n-n\pi$

9.10.11

$$\begin{aligned}\text{ber}'_\nu x&=\tfrac{1}{2}M_{\nu+1}\cos\,(\theta_{\nu+1}-\tfrac{1}{4}\pi)-\tfrac{1}{2}M_{\nu-1}\cos\,(\theta_{\nu-1}-\tfrac{1}{4}\pi)\\&=(\nu/x)M_\nu\cos\theta_\nu+M_{\nu+1}\cos\,(\theta_{\nu+1}-\tfrac{1}{4}\pi)\\&=-(\nu/x)M_\nu\cos\theta_\nu-M_{\nu-1}\cos\,(\theta_{\nu-1}-\tfrac{1}{4}\pi)\end{aligned}$$

9.10.12

$$\begin{aligned}\text{bei}'_\nu x&=\tfrac{1}{2}M_{\nu+1}\sin\,(\theta_{\nu+1}-\tfrac{1}{4}\pi)-\tfrac{1}{2}M_{\nu-1}\sin\,(\theta_{\nu-1}-\tfrac{1}{4}\pi)\\&=(\nu/x)M_\nu\sin\theta_\nu+M_{\nu+1}\sin\,(\theta_{\nu+1}-\tfrac{1}{4}\pi)\\&=-(\nu/x)M_\nu\sin\theta_\nu-M_{\nu-1}\sin\,(\theta_{\nu-1}-\tfrac{1}{4}\pi)\end{aligned}$$

9.10.13

$$\text{ber}'\,x=M_1\cos\,(\theta_1-\tfrac{1}{4}\pi),\qquad \text{bei}'\,x=M_1\sin\,(\theta_1-\tfrac{1}{4}\pi)$$

9.10.14

$$\begin{aligned}M'_\nu&=(\nu/x)M_\nu+M_{\nu+1}\cos\,(\theta_{\nu+1}-\theta_\nu-\tfrac{1}{4}\pi)\\&=-(\nu/x)M_\nu-M_{\nu-1}\cos\,(\theta_{\nu-1}-\theta_\nu-\tfrac{1}{4}\pi)\end{aligned}$$

9.10.15

$$\begin{aligned}\theta'_\nu&=(M_{\nu+1}/M_\nu)\sin\,(\theta_{\nu+1}-\theta_\nu-\tfrac{1}{4}\pi)\\&=-(M_{\nu-1}/M_\nu)\sin\,(\theta_{\nu-1}-\theta_\nu-\tfrac{1}{4}\pi)\end{aligned}$$

[6] The coefficients of these terms given in [9.17] are incorrect. The present results are due to Mr. G. F. Miller.

9.10.16

$$\begin{aligned}M'_0&=M_1\cos\,(\theta_1-\theta_0-\tfrac{1}{4}\pi)\\\theta'_0&=(M_1/M_0)\sin\,(\theta_1-\theta_0-\tfrac{1}{4}\pi)\end{aligned}$$

9.10.17

$$d(xM_\nu^2\theta'_\nu)/dx=xM_\nu^2,\qquad x^2M''_\nu+xM'_\nu-\nu^2M_\nu=x^2M_\nu\theta'^2_\nu$$

9.10.18

$$N_\nu=\sqrt{(\text{ker}_\nu^2 x+\text{kei}_\nu^2 x)},\qquad \phi_\nu=\arctan\,(\text{kei}_\nu x/\text{ker}_\nu x)$$

9.10.19 $\quad \text{ker}_\nu x=N_\nu\cos\phi_\nu,\qquad \text{kei}_\nu x=N_\nu\sin\phi_\nu$

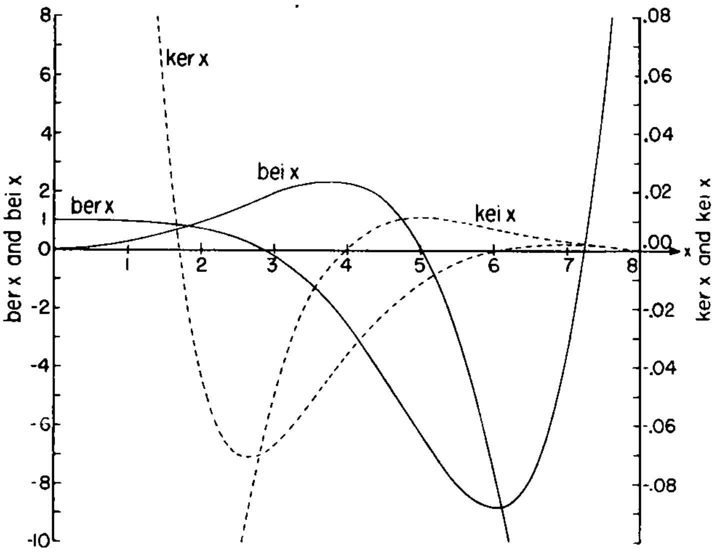

FIGURE 9.10. ber x, bei x, ker x *and* kei x.

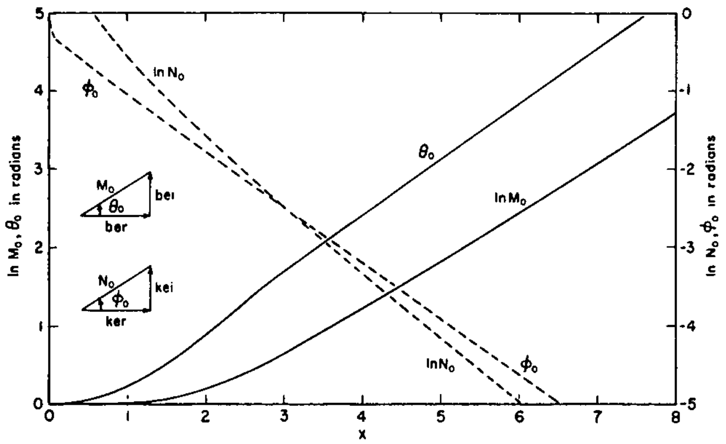

FIGURE 9.11. ln $M_0(x)$, $\theta_0(x)$, ln $N_0(x)$ *and* $\phi_0(x)$.

Equations **9.10.11** to **9.10.17** hold with the symbols b, M, θ replaced throughout by k, N, ϕ, respectively. In place of **9.10.10**

9.10.20 $\quad N_{-\nu}=N_\nu,\qquad \phi_{-\nu}=\phi_\nu+\nu\pi$