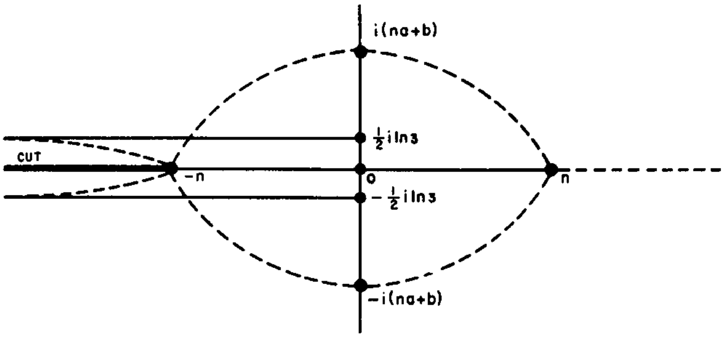

FIGURE 9.5. *Zeros of* $Y_n(z)$ *and* $Y_n'(z)$ . . . $|\arg z|\leq\pi$.

Figure 9.5 shows the approximate distribution of the complex zeros of $Y_n(z)$ in the region $|\arg z|\leq\pi$. The figure is symmetrical about the real axis. The two curves on the left extend to infinity, having the asymptotes

$$\mathscr{I}z=\pm\tfrac{1}{2}\ln 3=\pm.54931\ldots$$

There are an infinite number of zeros near each of these curves.

The two curves extending from $z=-n$ to $z=n$ and bounding an eye-shaped domain intersect the imaginary axis at the points $\pm i(na+b)$, where

$$a=\sqrt{t_0^2-1}=.66274\ldots$$

$$b=\tfrac{1}{2}\sqrt{1-t_0^{-2}}\ln 2=.19146\ldots$$

and $t_0=1.19968\ldots$ is the positive root of $\coth t=t$. There are n zeros near each of these curves. Asymptotic expansions of these zeros for large n are given by the right of **9.5.22** with $\nu=n$ and $\zeta=n^{-2/3}\beta_s$ or $n^{-2/3}\bar{\beta}_s$, where β_s, $\bar{\beta}_s$ are the complex zeros of $\mathrm{Bi}(z)$ (see **10.4**).

Figure 9.5 is also applicable to the zeros of $Y_n'(z)$. There are again an infinite number near the infinite curves, and n near each of the finite curves. Asymptotic expansions of the latter for large n are given by the right of **9.5.24** with $\nu=n$ and $\zeta=n^{-2/3}\beta_s'$ or $n^{-2/3}\bar{\beta}_s'$; where β_s' and $\bar{\beta}_s'$ are the complex zeros of $\mathrm{Bi}'(z)$.

Numerical values of the three smallest complex zeros of $Y_0(z)$, $Y_1(z)$ and $Y_1'(z)$ in the region $0<\arg z<\pi$ are given below.

For further details see [9.36] and [9.13]. The latter reference includes tables to facilitate computation.

Complex Zeros of the Hankel Functions

The approximate distribution of the zeros of $H_n^{(1)}(z)$ and its derivative in the region $|\arg z|\leq\pi$ is indicated in a similar manner on **Figure 9.6.**

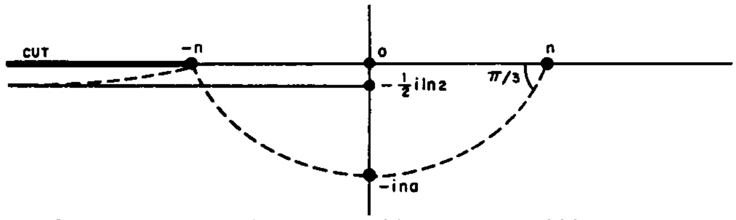

FIGURE 9.6. *Zeros of* $H_n^{(1)}(z)$ *and* $H_n^{(1)\prime}(z)$ . . . $|\arg z|\leq\pi$.

The asymptote of the solitary infinite curve is given by

$$\mathscr{I}z=-\tfrac{1}{2}\ln 2=-.34657\ldots$$

Zeros of $Y_0(z)$ *and Values of* $Y_1(z)$ *at the Zeros*[3]

Zero		Y_1	
Real	*Imag.*	*Real*	*Imag.*
−2.40301 6632	+.53988 2313	+.10074 7689	−.88196 7710
−5.51987 6702	+.54718 0011	−.02924 6418	+.58716 9503
−8.65367 2403	+.54841 2067	+.01490 8063	−.46945 8752

Zeros of $Y_1(z)$ *and Values of* $Y_0(z)$ *at the Zeros*

Zero		Y_0	
Real	*Imag.*	*Real*	*Imag.*
−0.50274 3273	+.78624 3714	−.45952 7684	+1.31710 1937
−3.83353 5193	+.56235 6538	+.04830 1909	−0.69251 2884
−7.01590 3683	+.55339 3046	−.02012 6949	+0.51864 2833

Zeros of $Y_1'(z)$ *and Values of* $Y_1(z)$ *at the Zeros*

Zero		Y_1	
Real	*Imag.*	*Real*	*Imag.*
+0.57678 5129	+.90398 4792	−.76349 7088	+.58924 4865
−1.94047 7342	+.72118 5919	+.16206 4006	−.95202 7886
−5.33347 8617	+.56721 9637	−.03179 4008	+.59685 3673

[3] From National Bureau of Standards, Tables of the Bessel functions $Y_0(z)$ and $Y_1(z)$ for complex arguments, Columbia Univ. Press, New York, N.Y., 1950 (with permission).