McMahon's Expansions for Large Zeros

When ν is fixed, $s>>\nu$ and $\mu=4\nu^2$

9.5.12

$$j_{\nu,s}, y_{\nu,s}\sim\beta-\frac{\mu-1}{8\beta}-\frac{4(\mu-1)(7\mu-31)}{3(8\beta)^3}-\frac{32(\mu-1)(83\mu^2-982\mu+3779)}{15(8\beta)^5}-\frac{64(\mu-1)(6949\mu^3-1\ 53855\mu^2+15\ 85743\mu-62\ 77237)}{105(8\beta)^7}-\dots$$

where $\beta=(s+\frac{1}{2}\nu-\frac{1}{4})\pi$ for $j_{\nu,s}$, $\beta=(s+\frac{1}{2}\nu-\frac{3}{4})\pi$ for $y_{\nu,s}$. With $\beta=(t+\frac{1}{2}\nu-\frac{1}{4})\pi$, the right of **9.5.12** is the asymptotic expansion of $\rho_\nu(t)$ for large t.

9.5.13

$$j'_{\nu,s}, y'_{\nu,s}\sim\beta'-\frac{\mu+3}{8\beta'}-\frac{4(7\mu^2+82\mu-9)}{3(8\beta')^3}-\frac{32(83\mu^3+2075\mu^2-3039\mu+3537)}{15(8\beta')^5}-\frac{64(6949\mu^4+2\ 96492\mu^3-12\ 48002\mu^2+74\ 14380\mu-58\ 53627)}{105(8\beta')^7}-\dots$$

where $\beta'=(s+\frac{1}{2}\nu-\frac{3}{4})\pi$ for $j'_{\nu,s}$, $\beta'=(s+\frac{1}{2}\nu-\frac{1}{4})\pi$ for $y'_{\nu,s}$, $\beta'=(t+\frac{1}{2}\nu+\frac{1}{4})\pi$ for $\sigma_\nu(t)$. For higher terms in **9.5.12** and **9.5.13** see [9.4] or [9.40].

Asymptotic Expansions of Zeros and Associated Values for Large Orders

9.5.14

$$j_{\nu,1}\sim\nu+1.85575\ 71\nu^{1/3}+1.03315\ 0\nu^{-1/3}-.00397\nu^{-1}-.0908\nu^{-5/3}+.043\nu^{-7/3}+\dots$$

9.5.15

$$y_{\nu,1}\sim\nu+.93157\ 68\nu^{1/3}+.26035\ 1\nu^{-1/3}+.01198\nu^{-1}-.0060\nu^{-5/3}-.001\nu^{-7/3}+\dots$$

9.5.16

$$j'_{\nu,1}\sim\nu+.80861\ 65\nu^{1/3}+.07249\ 0\nu^{-1/3}-.05097\nu^{-1}+.0094\nu^{-5/3}+\dots$$

9.5.17

$$y'_{\nu,1}\sim\nu+1.82109\ 80\nu^{1/3}+.94000\ 7\nu^{-1/3}-.05808\nu^{-1}-.0540\nu^{-5/3}+\dots$$

9.5.18

$$J'_\nu(j_{\nu,1})\sim-1.11310\ 28\nu^{-2/3}/(1+1.48460\ 6\nu^{-2/3}+.43294\nu^{-4/3}-.1943\nu^{-2}+.019\nu^{-8/3}+\dots)$$

9.5.19

$$Y'_\nu(y_{\nu,1})\sim.95554\ 86\nu^{-2/3}/(1+.74526\ 1\nu^{-2/3}+.10910\nu^{-4/3}-.0185\nu^{-2}-.003\nu^{-8/3}+\dots)$$

9.5.20

$$J_\nu(j'_{\nu,1})\sim.67488\ 51\nu^{-1/3}(1-.16172\ 3\nu^{-2/3}+.02918\nu^{-4/3}-.0068\nu^{-2}+\dots)$$

9.5.21

$$Y_\nu(y'_{\nu,1})\sim.57319\ 40\nu^{-1/3}(1-.36422\ 0\nu^{-2/3}+.09077\nu^{-4/3}+.0237\nu^{-2}+\dots)$$

Corresponding expansions for $s=2$, 3 are given in [9.40]. These expansions become progressively weaker as s increases; those which follow do not suffer from this defect.

Uniform Asymptotic Expansions of Zeros and Associated Values for Large Orders

9.5.22 $$j_{\nu,s}\sim\nu z(\zeta)+\sum_{k=1}^{\infty}\frac{f_k(\zeta)}{\nu^{2k-1}}\quad\text{with } \zeta=\nu^{-2/3}a_s$$

9.5.23

$$J'_\nu(j_{\nu,s})\sim-\frac{2}{\nu^{2/3}}\frac{\mathrm{Ai}'(a_s)}{z(\zeta)h(\zeta)}\left\{1+\sum_{k=1}^{\infty}\frac{F_k(\zeta)}{\nu^{2k}}\right\}\quad\text{with } \zeta=\nu^{-2/3}a_s$$

9.5.24 $$j'_{\nu,s}\sim\nu z(\zeta)+\sum_{k=1}^{\infty}\frac{g_k(\zeta)}{\nu^{2k-1}}\quad\text{with } \zeta=\nu^{-2/3}a'_s$$

9.5.25

$$J_\nu(j'_{\nu,s})\sim\mathrm{Ai}(a'_s)\frac{h(\zeta)}{\nu^{1/3}}\left\{1+\sum_{k=1}^{\infty}\frac{G_k(\zeta)}{\nu^{2k}}\right\}\quad\text{with } \zeta=\nu^{-2/3}a'_s$$

where a_s, a'_s are the sth negative zeros of $\mathrm{Ai}(z)$, $\mathrm{Ai}'(z)$ (see **10.4**), $z=z(\zeta)$ is the inverse function defined implicitly by **9.3.39**, and

9.5.26

$$h(\zeta)=\{4\zeta/(1-z^2)\}^{\frac{1}{4}}$$
$$f_1(\zeta)=\tfrac{1}{2}z(\zeta)\{h(\zeta)\}^2b_0(\zeta)$$
$$g_1(\zeta)=\tfrac{1}{2}\zeta^{-1}z(\zeta)\{h(\zeta)\}^2c_0(\zeta)$$

where $b_0(\zeta)$, $c_0(\zeta)$ appear in **9.3.42** and **9.3.46.** Tables of the leading coefficients follow. More extensive tables are given in [9.40].

The expansions of $y_{\nu,s}$, $Y'_\nu(y_{\nu,s})$, $y'_{\nu,s}$ and $Y_\nu(y'_{\nu,s})$ corresponding to **9.5.22** to **9.5.25** are obtained by changing the symbols j, J, Ai, Ai′, a_s and a'_s to y, Y, −Bi, −Bi′, b_s and b'_s respectively.