8.14.8 $\int_{-1}^{1} P_\nu(x)Q_\rho(x)dx=[(\nu-\rho)(\rho+\nu+1)]^{-1}\left\{1-\cos(\rho\pi-\nu\pi)-\frac{2}{\pi}\sin\pi\nu\cos\pi\nu[\psi(\nu+1)-\psi(\rho+1)]\right\}$

$(\mathscr{R}\nu>0,\ \mathscr{R}\rho>0,\ \rho\neq\nu)$

8.14.9 $\int_{-1}^{1} P_\nu(x)Q_\nu(x)dx=-\frac{1}{\pi}(2\nu+1)^{-1}\sin 2\nu\pi\psi'(\nu+1)$ $(\mathscr{R}\nu>0)$

(m, n, l positive integers)

8.14.10

$$\int_{-1}^{1} Q_n^m(x)P_l^m(x)dx=(-1)^m\frac{1-(-1)^{l+n}(n+m)!}{(l-n)(l+n+1)(n-m)!}$$

8.14.11 $\int_{-1}^{1} P_n^m(x)P_l^m(x)dx=0 \qquad (l\neq n)$

8.14.12 $\int_{-1}^{1} P_n^m(x)P_n^l(x)(1-x^2)^{-1}dx=0 \qquad (l\neq m)$

8.14.13 $\int_{-1}^{1}[P_n^m(x)]^2dx=(n+\tfrac{1}{2})^{-1}(n+m)!/(n-m)!$

8.14.14

$$\int_{-1}^{1}(1-x^2)^{-1}[P_n^m(x)]^2dx=(n+m)!/m(n-m)!$$

8.14.15

$$\int_0^1 P_\nu(x)x^\rho dx=\frac{\pi^{\frac{1}{2}}2^{-\rho-1}\Gamma(1+\rho)}{\Gamma(1+\frac{1}{2}\rho-\frac{1}{2}\nu)\Gamma(\frac{1}{2}\rho+\frac{1}{2}\nu+\frac{3}{2})}$$

$(\mathscr{R}\rho>-1)$

8.14.16

$$\int_0^\pi(\sin t)^{\alpha-1}P_\nu^{-\mu}(\cos t)dt=\frac{2^{-\mu}\pi\Gamma(\frac{1}{2}\alpha+\frac{1}{2}\mu)\Gamma(\frac{1}{2}\alpha-\frac{1}{2}\mu)}{\Gamma(\frac{1}{2}+\frac{1}{2}\alpha+\frac{1}{2}\nu)\Gamma(\frac{1}{2}\alpha-\frac{1}{2}\nu)\Gamma(\frac{1}{2}\mu+\frac{1}{2}\nu+1)\Gamma(\frac{1}{2}\mu-\frac{1}{2}\nu+\frac{1}{2})} \qquad (\mathscr{R}(\alpha\pm\mu)>0)$$

8.14.17

$$P_\nu^{-m}(z)=(z^2-1)^{-\frac{1}{2}m}\int_1^z\cdots\int_1^z P_\nu(z)(dz)^m$$

8.14.18

$$Q_\nu^{-m}(z)=(-1)^m(z^2-1)^{-\frac{1}{2}m}\int_z^\infty\cdots\int_z^\infty Q_\nu(z)(dz)^m$$

For other integrals, see [8.2], [8.4] and chapter **22.**

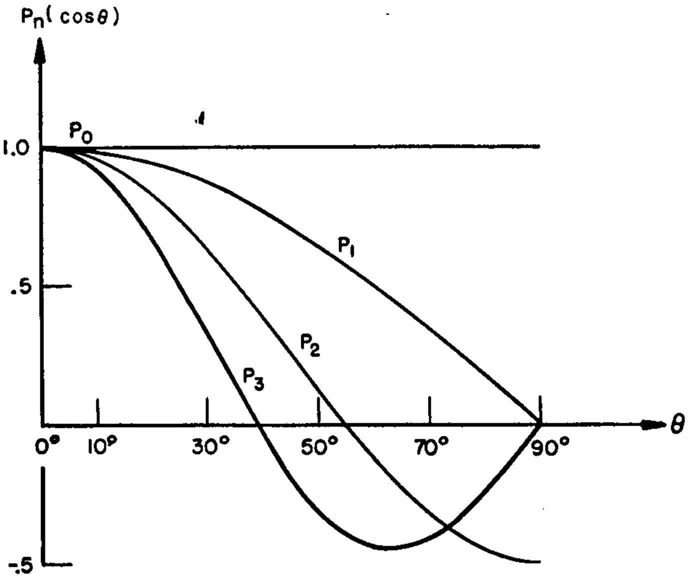

FIGURE 8.1. $P_n(\cos\theta)$. $n=0(1)3$.

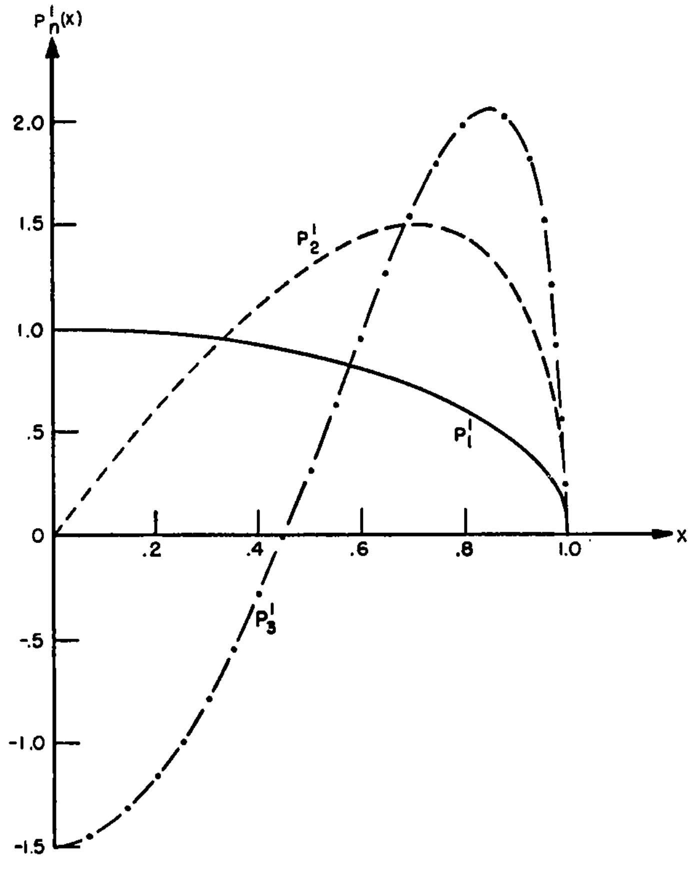

FIGURE 8.2. $P_n^1(x)$. $n=1(1)3$, $x\leq 1$.