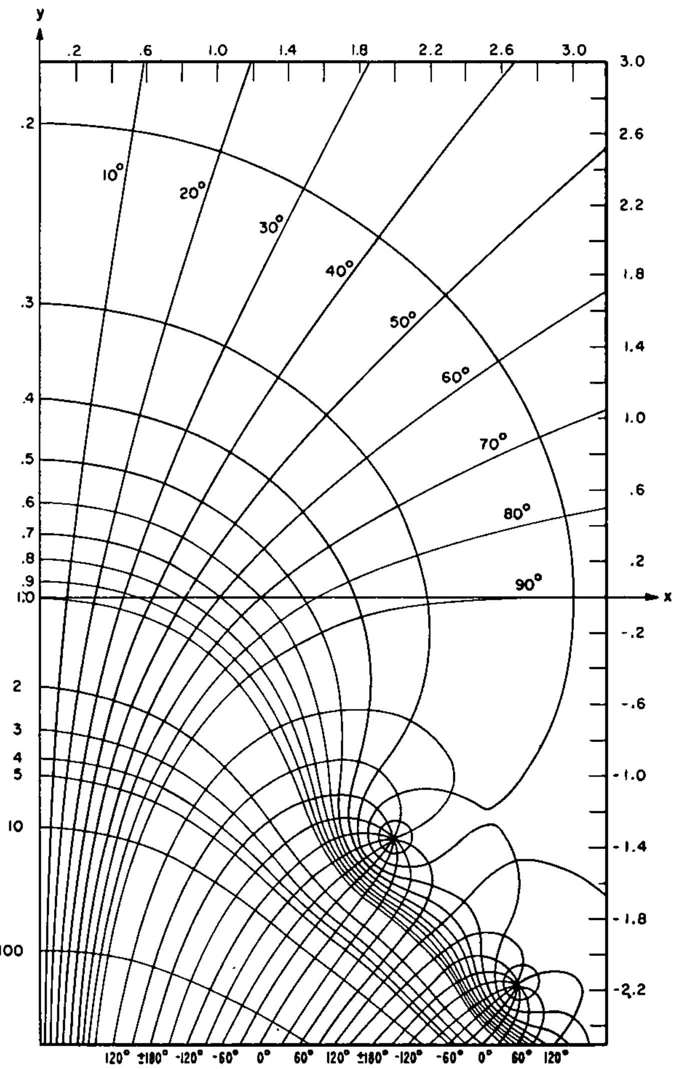

FIGURE 7.3. *Altitude Chart of $w(z)$.*

Inequalities [7.11], [7.17]

7.1.13

$$\frac{1}{x+\sqrt{x^2+2}}<e^{x^2}\int_x^\infty e^{-t^2}dt\leq\frac{1}{x+\sqrt{x^2+\frac{4}{\pi}}}\qquad(x\geq 0)$$

(For other inequalities see [7.2].)

Continued Fractions

7.1.14

$$2e^{z^2}\int_z^\infty e^{-t^2}dt=\frac{1}{z+}\,\frac{1/2}{z+}\,\frac{1}{z+}\,\frac{3/2}{z+}\,\frac{2}{z+}\cdots\quad(\mathscr{R}z>0)$$

7.1.15

$$\frac{1}{\sqrt{\pi}}\int_{-\infty}^\infty\frac{e^{-t^2}dt}{z-t}=\frac{1}{z-}\,\frac{1/2}{z-}\,\frac{1}{z-}\,\frac{3/2}{z-}\,\frac{2}{z-}\cdots$$
$$=\frac{1}{\sqrt{\pi}}\lim_{n\to\infty}\sum_{k=1}^{n}\frac{H_k^{(n)}}{z-x_k^{(n)}}\qquad(\mathscr{I}z\neq 0)$$

$x_k^{(n)}$ and $H_k^{(n)}$ are the zeros and weight factors of the Hermite polynomials. For numerical values see chapter **25**.

Value at Infinity

7.1.16 $\operatorname{erf} z\to 1\left(z\to\infty \text{ in } |\arg z|<\frac{\pi}{4}\right)$

Maximum and Inflection Points for Dawson's Integral [7.31]

$$F(x)=e^{-x^2}\int_0^x e^{t^2}dt$$

7.1.17 $F(.92413\ 88730\ldots)=.54104\ 42246\ldots$

7.1.18 $F(1.50197\ 52682\ldots)=.42768\ 66160\ldots$

Derivatives

7.1.19

$$\frac{d^{n+1}}{dz^{n+1}}\operatorname{erf} z=(-1)^n\frac{2}{\sqrt{\pi}}H_n(z)e^{-z^2}\qquad(n=0,1,2,\ldots)$$

7.1.20

$$w^{(n+2)}(z)+2zw^{(n+1)}(z)+2(n+1)w^{(n)}(z)=0\qquad(n=0,1,2,\ldots)$$

$$w^{(0)}(z)=w(z),\qquad w'(z)=-2zw(z)+\frac{2i}{\sqrt{\pi}}$$

(For the Hermite polynomials $H_n(z)$ see chapter **22.**)

Relation to Confluent Hypergeometric Function (see chapter 13)

7.1.21

$$\operatorname{erf} z=\frac{2z}{\sqrt{\pi}}M\left(\frac{1}{2},\frac{3}{2},-z^2\right)=\frac{2z}{\sqrt{\pi}}e^{-z^2}M\left(1,\frac{3}{2},z^2\right)$$

The Normal Distribution Function With Mean *m* and Standard Deviation σ (see chapter 26)

7.1.22 $$\frac{1}{\sigma\sqrt{2\pi}}\int_{-\infty}^x e^{-\frac{(t-m)^2}{2\sigma^2}}dt=\frac{1}{2}\left(1+\operatorname{erf}\left(\frac{x-m}{\sigma\sqrt{2}}\right)\right)$$

Asymptotic Expansion

7.1.23

$$\sqrt{\pi}ze^{z^2}\operatorname{erfc} z\sim 1+\sum_{m=1}^\infty(-1)^m\frac{1\cdot 3\ldots(2m-1)}{(2z^2)^m}$$
$$\left(z\to\infty,\ |\arg z|<\frac{3\pi}{4}\right)$$