6.5.5

Probability Integral of the χ^2-Distribution

$$P(\chi^2|\nu)=\frac{1}{2^{\frac{1}{2}\nu}\Gamma\left(\frac{\nu}{2}\right)}\int_0^{\chi^2} t^{\frac{1}{2}\nu-1}e^{-\frac{t}{2}}\,dt$$

6.5.6

(Pearson's Form of the Incomplete Gamma Function)

$$I(u,p)=\frac{1}{\Gamma(p+1)}\int_0^{u\sqrt{p+1}} e^{-t}t^p\,dt$$

$$=P(p+1,u\sqrt{p+1})$$

6.5.7 $$C(x,a)=\int_x^\infty t^{a-1}\cos t\,dt \qquad (\mathscr{R}a<1)$$

6.5.8 $$S(x,a)=\int_x^\infty t^{a-1}\sin t\,dt \qquad (\mathscr{R}a<1)$$

6.5.9

$$E_n(x)=\int_1^\infty e^{-xt}t^{-n}\,dt=x^{n-1}\Gamma(1-n,x)$$

6.5.10

$$\alpha_n(x)=\int_1^\infty e^{-xt}t^n\,dt=x^{-n-1}\Gamma(1+n,x)$$

6.5.11 $$e_n(x)=\sum_{j=0}^n \frac{x^j}{j!}$$

Incomplete Gamma Function as a Confluent Hypergeometric Function (see chapter 13)

6.5.12 $$\gamma(a,x)=a^{-1}x^a e^{-x}M(1,1+a,x)$$

$$=a^{-1}x^a M(a,1+a,-x)$$

Special Values

6.5.13

$$P(n,x)=1-\left(1+x+\frac{x^2}{2!}+\ldots+\frac{x^{n-1}}{(n-1)!}\right)e^{-x}$$

$$=1-e_{n-1}(x)e^{-x}$$

For relation to the Poisson distribution, see 26.4.

6.5.14 $$\gamma^*(-n,x)=x^n$$

6.5.15 $$\Gamma(0,x)=\int_x^\infty e^{-t}t^{-1}\,dt=E_1(x)$$

6.5.16 $$\gamma(\tfrac{1}{2},x^2)=2\int_0^x e^{-t^2}\,dt=\sqrt{\pi}\operatorname{erf} x$$

6.5.17 $$\Gamma(\tfrac{1}{2},x^2)=2\int_x^\infty e^{-t^2}dt=\sqrt{\pi}\operatorname{erfc} x$$

6.5.18 $$\tfrac{1}{2}\sqrt{\pi}\,x\,\gamma^*(\tfrac{1}{2},-x^2)=\int_0^x e^{t^2}dt$$

6.5.19 $$\Gamma(-n,x)=\frac{(-1)^n}{n!}\left[E_1(x)-e^{-x}\sum_{j=0}^{n-1}\frac{(-1)^j j!}{x^{j+1}}\right]$$

6.5.20 $$\Gamma(a,ix)=e^{\frac{1}{2}\pi ia}\,[C(x,a)-iS(x,a)]$$

Recurrence Formulas

6.5.21 $$P(a+1,x)=P(a,x)-\frac{x^a e^{-x}}{\Gamma(a+1)}$$

6.5.22 $$\gamma(a+1,x)=a\gamma(a,x)-x^a e^{-x}$$

6.5.23 $$\gamma^*(a-1,x)=x\gamma^*(a,x)+\frac{e^{-x}}{\Gamma(a)}$$

Derivatives and Differential Equations

6.5.24

$$\left(\frac{\partial\gamma^*}{\partial\alpha}\right)_{\alpha=0}=-\int_x^\infty\frac{e^{-t}dt}{t}-\ln x=-E_1(x)-\ln x$$

6.5.25 $$\frac{\partial\gamma(a,x)}{\partial x}=-\frac{\partial\Gamma(a,x)}{\partial x}=x^{a-1}e^{-x}$$

6.5.26

$$\frac{\partial^n}{\partial x^n}[x^{-a}\Gamma(a,x)]=(-1)^n x^{-a-n}\Gamma(a+n,x) \qquad (n=0,1,2,\ldots)$$

6.5.27

$$\frac{\partial^n}{\partial x^n}[e^x x^a\gamma^*(a,x)]=e^x x^{a-n}\gamma^*(a-n,x) \qquad (n=0,1,2,\ldots)$$

6.5.28 $$x\frac{\partial^2\gamma^*}{\partial x^2}+(a+1+x)\frac{\partial\gamma^*}{\partial x}+a\gamma^*=0$$

Series Developments

6.5.29

$$\gamma^*(a,z)=e^{-z}\sum_{n=0}^\infty\frac{z^n}{\Gamma(a+n+1)}=\frac{1}{\Gamma(a)}\sum_{n=0}^\infty\frac{(-z)^n}{(a+n)n!} \qquad (|z|<\infty)$$