

Derivatives

$$5.1.26 \quad \frac{dE_n(z)}{dz} = -E_{n-1}(z) \quad (n=1, 2, 3, \dots)$$

5.1.27

$$\frac{d^n}{dz^n} [e^z E_1(z)] = \frac{d^{n-1}}{dz^{n-1}} [e^z E_1(z)] + \frac{(-1)^n (n-1)!}{z^n} \quad (n=1, 2, 3, \dots)$$

Definite and Indefinite Integrals

(For more extensive tables of integrals see [5.3], [5.6], [5.11], [5.12], [5.13]. For integrals involving $E_n(x)$ see [5.9].)

$$5.1.28 \quad \int_0^\infty \frac{e^{-at}}{b+t} dt = e^{ab} E_1(ab)$$

5.1.29

$$\int_0^\infty \frac{e^{iat}}{b+t} dt = e^{-iab} E_1(-iab) \quad (a > 0, b > 0)$$

5.1.30

$$\int_0^\infty \frac{t-ib}{t^2+b^2} e^{iat} dt = e^{ab} E_1(ab) \quad (a > 0, b > 0)$$

5.1.31

$$\int_0^\infty \frac{t+ib}{t^2+b^2} e^{iat} dt = e^{-ab} (-\text{Ei}(ab) + i\pi) \quad (a > 0, b > 0)$$

$$5.1.32 \quad \int_0^\infty \frac{e^{-at} - e^{-bt}}{t} dt = \ln \frac{b}{a}$$

$$5.1.33 \quad \int_0^\infty E_1^2(t) dt = 2 \ln 2$$

5.1.34

$$\int_0^\infty e^{-at} E_n(t) dt = \frac{(-1)^{n-1}}{a^n} \left[\ln(1+a) + \sum_{k=1}^{n-1} \frac{(-1)^k a^k}{k} \right] \quad (a > -1)$$

5.1.35

$$\int_0^1 \frac{e^{at} \sin bt}{t} dt = \pi - \arctan \frac{b}{a} + \mathcal{S} E_1(-a+ib) \quad (a > 0, b > 0)$$

5.1.36

$$\int_0^1 \frac{e^{-at} \sin bt}{t} dt = \arctan \frac{b}{a} + \mathcal{S} E_1(a+ib) \quad (a > 0, b \text{ real})$$

5.1.37

$$\int_0^1 \frac{e^{at}(1-\cos bt)}{t} dt = \frac{1}{2} \ln \left(1 + \frac{b^2}{a^2} \right) + \text{Ei}(a) + \mathcal{R} E_1(-a+ib) \quad (a > 0, b \text{ real})$$

5.1.38

$$\int_0^1 \frac{e^{-at}(1-\cos bt)}{t} dt = \frac{1}{2} \ln \left(1 + \frac{b^2}{a^2} \right) - E_1(a) + \mathcal{R} E_1(a+ib) \quad (a > 0, b \text{ real})$$

$$5.1.39 \quad \int_0^z \frac{1-e^{-t}}{t} dt = E_1(z) + \ln z + \gamma$$

$$5.1.40 \quad \int_0^x \frac{e^t-1}{t} dt = \text{Ei}(x) - \ln x - \gamma \quad (x > 0)$$

5.1.41

$$\int \frac{e^{ix}}{a^2+x^2} dx = \frac{i}{2a} [e^{-a} E_1(-a-ix) - e^a E_1(a-ix)] + \text{const.}$$

5.1.42

$$\int \frac{x e^{ix}}{a^2+x^2} dx = -\frac{1}{2} [e^{-a} E_1(-a-ix) + e^a E_1(a-ix)] + \text{const.}$$

5.1.43

$$\int \frac{e^x}{a^2+x^2} dx = -\frac{1}{a} \mathcal{S}(e^{ia} E_1(-x+ia)) + \text{const.} \quad (a > 0)$$

5.1.44

$$\int \frac{x e^x}{a^2+x^2} dx = -\mathcal{R}(e^{ia} E_1(-x+ia)) + \text{const.} \quad (a > 0)$$

Relation to Incomplete Gamma Function (see 6.5)

$$5.1.45 \quad E_n(z) = z^{n-1} \Gamma(1-n, z)$$

$$5.1.46 \quad \alpha_n(z) = z^{-n-1} \Gamma(n+1, z)$$

$$5.1.47 \quad \beta_n(z) = z^{-n-1} [\Gamma(n+1, -z) - \Gamma(n+1, z)]$$

Relation to Spherical Bessel Functions (see 10.2)

$$5.1.48 \quad \alpha_0(z) = \sqrt{\frac{2}{\pi z}} K_{\frac{1}{2}}(z), \quad \beta_0(z) = \sqrt{\frac{2\pi}{z}} I_{\frac{1}{2}}(z)$$

$$5.1.49 \quad \alpha_1(z) = \sqrt{\frac{2}{\pi z}} K_{3/2}(z), \quad \beta_1(z) = -\sqrt{\frac{2\pi}{z}} I_{3/2}(z)$$