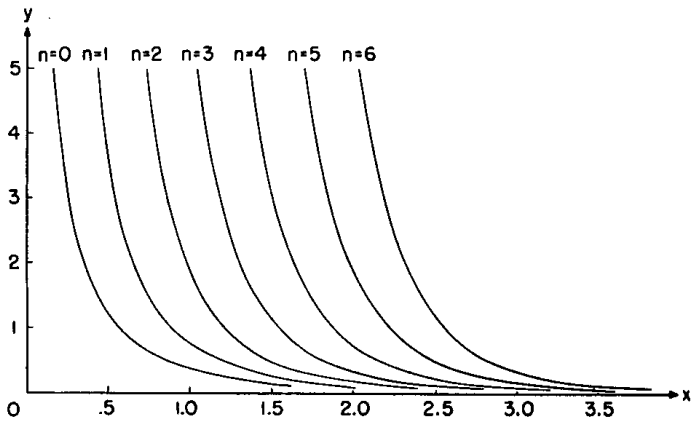

FIGURE 5.3. $y=\alpha_n(x)$
$n=0(1)6$

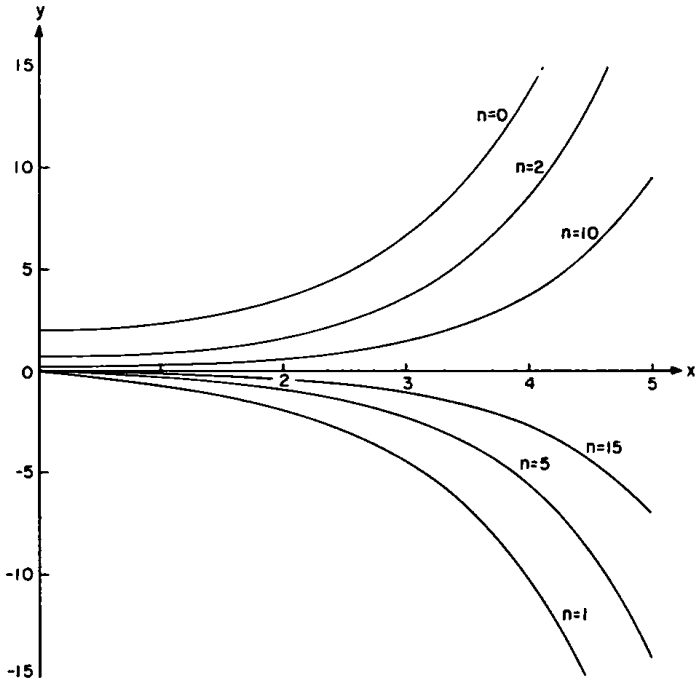

FIGURE 5.4. $y=\beta_n(x)$
$n=0, 1, 2, 5, 10, 15$

Series Expansions

5.1.10 $$\mathrm{Ei}(x)=\gamma+\ln x+\sum_{n=1}^{\infty}\frac{x^n}{nn!} \qquad (x>0)$$

5.1.11

$$E_1(z)=-\gamma-\ln z-\sum_{n=1}^{\infty}\frac{(-1)^n z^n}{nn!} \qquad (|\arg z|<\pi)$$

5.1.12

$$E_n(z)=\frac{(-z)^{n-1}}{(n-1)!}[-\ln z+\psi(n)]-\sum_{\substack{m=0\\ m\neq n-1}}^{\infty}{}' \frac{(-z)^m}{(m-n+1)m!} \qquad (|\arg z|<\pi)$$

$$\psi(1)=-\gamma,\ \psi(n)=-\gamma+\sum_{m=1}^{n-1}\frac{1}{m} \qquad (n>1)$$

$\gamma=.57721\ 56649\ \ldots$ is Euler's constant.

Symmetry Relation

5.1.13 $$E_n(\bar{z})=\overline{E_n(z)}$$

Recurrence Relations

5.1.14

$$E_{n+1}(z)=\frac{1}{n}[e^{-z}-zE_n(z)] \qquad (n=1,2,3,\ldots)$$

5.1.15 $$z\alpha_n(z)=e^{-z}+n\alpha_{n-1}(z) \qquad (n=1,2,3,\ldots)$$

5.1.16

$$z\beta_n(z)=(-1)^n e^z-e^{-z}+n\beta_{n-1}(z) \qquad (n=1,2,3,\ldots)$$

Inequalities [5.8], [5.4]

5.1.17

$$\frac{n-1}{n}E_n(x)<E_{n+1}(x)<E_n(x) \qquad (x>0; n=1,2,3,\ldots)$$

5.1.18

$$E_n^2(x)<E_{n-1}(x)E_{n+1}(x) \qquad (x>0; n=1,2,3,\ldots)$$

5.1.19

$$\frac{1}{x+n}<e^x E_n(x)\leq\frac{1}{x+n-1} \qquad (x>0; n=1,2,3,\ldots)$$

5.1.20

$$\tfrac{1}{2}\ln\left(1+\frac{2}{x}\right)<e^x E_1(x)<\ln\left(1+\frac{1}{x}\right) \qquad (x>0)$$

5.1.21

$$\frac{d}{dx}\left[\frac{E_n(x)}{E_{n-1}(x)}\right]>0 \qquad (x>0; n=1,2,3,\ldots)$$

Continued Fraction

5.1.22

$$E_n(z)=e^{-z}\left(\frac{1}{z+}\frac{n}{1+}\frac{1}{z+}\frac{n+1}{1+}\frac{2}{z+}\cdots\right) \qquad (|\arg z|<\pi)$$

Special Values

5.1.23 $$E_n(0)=\frac{1}{n-1} \qquad (n>1)$$

5.1.24 $$E_0(z)=\frac{e^{-z}}{z}$$

5.1.25 $$\alpha_0(z)=\frac{e^{-z}}{z},\ \beta_0(z)=\frac{2}{z}\sinh z$$