Double Precision Multiplication and Division on a Desk Calculator

Example 7. Multiply M=20243 97459 71664 32102 by m=69732 82428 43662 95023 on a 10×10×20 desk calculating machine.

Let M_0=20243 97459, M_1=71664 32102, m_0= 69732 82428, m_1=43662 95023. Then $Mm=M_0m_010^{20}+(M_0m_1+M_1m_0)10^{10}+M_1m_1$.

(1) Multiply M_1m_1=31290 75681 96300 28346 and record the digits 96300 28346 appearing in positions 1 to 10 of the product dial.

(2) Transfer the digits 31290 75681 from positions 11 to 20 of the product dial to positions 1 to 10 of the product dial.

(3) Multiply cumulatively $M_1m_0+M_0m_1$+31290 75681=58812 67160 12663 25894 and record the digits 12663 25894 in positions 1 to 10.

(4) Transfer the digits 58812 67160 from positions 11 to 20 to positions 1 to 10.

(5) Multiply cumulatively M_0m_0+58812 67160 =14116 69523 40138 17612. The results as obtained are shown below,

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                                          96300 28346
                              12663 25894
14116 69523 40138 17612
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14116 69523 40138 17612 12663 25894 96300 28346
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If the product Mm is wanted to 20 digits, only the result obtained in step 5 need be recorded. Further, if the allowable error in the 20th place is a unit, the operation M_1m_1 may be omitted. When either of the factors M or m contains less than 20 digits it is convenient to position the numbers as if they both had 20 digits. This multiplication process may be extended to any higher accuracy desired.

Example 8. Divide N=14116 69523 40138 17612 by d=20243 97459 71664 32102.

Method (1)—*linear interpolation.*

N/20243 97459·10^{10}=.69732 82430 90519 39054
N/20243 97460·10^{10}=.69732 82427 46057 26941
Difference=3 44462 12113.

Difference ×.71664 32102=24685 64402$\underline{8}$·10^{-20} (note this is an 11×10 multiplication).

Quotient=
(69732 82430 90519 39054−246856 4402$\underline{8}$)·10^{-20}
=.69732 82428 43662 9502$\underline{6}$

There is an error of 3 units in the 20th place due to neglect of the contribution from second differences.

Method (2)—If N and d are numbers each not more than 19 digits let $N=N_1+N_010^9$, $d=d_1+d_010^9$ where N_0 and d_0 contain 10 digits and N_1 and d_1 not more than 9 digits. Then

$$\frac{N}{d}=\frac{N_010^9+N_1}{d_010^9+d_1}\approx\frac{1}{d_010^9}\left[N-\frac{N_0d_1}{d_0}\right]$$

Here

N=14116 69523 40138 1761,
d=20243 97459 71664 3210
N_0=14116 69523, d_0=20243 97459,
d_1=71664 3210

(1) N_0d_1=10116 63378 42188 8830 (product dial).

(2) $(N_0d_1)/d_0$=49973 55504 (quotient dial).

(3) $N-(N_0d_1)/d_0$=14116 69522 90164 62106 (product dial).

(4) $[N-(N_0d_1)/d_0]/d_010^9$=.69732 82428=first 10 digits of quotient in quotient dial. Remainder $=r$=08839 11654, in positions 1 to 10 of product dial.

(5) $r/(d_010^9)$=.43662 9502·10^{-10}=next 9 digits of quotient. N/d=.69732 82428 43662 9502. This method may be modified to give the quotient of 20 digit numbers. Method (1) may be extended to quotients of numbers containing more than 20 digits by employing higher order interpolation.

Example 9. Sum the series $S=1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\ldots$ to 5D using the Euler transform.

The sum of the first 8 terms is .634524 to 6D. If $u_n=1/n$ we get

n	u_n	Δu_n	$\Delta^2 u_n$	$\Delta^3 u_n$	$\Delta^4 u_n$
9	.111111				
		−11111			
10	.100000		2020		
		−9091		−505	
11	.090909		1515		156
		−7576		−349	
12	.083333		1166		
		−6410			
13	.076923				

From **3.6.27** we then obtain

$$S=.634524+\frac{.111111}{2}-\frac{(-.011111)}{2^2}+\frac{.002020}{2^3}-\frac{(-.000505)}{2^4}+\frac{.000156}{2^5}$$

$$=.634524+.055556+.002778+.000253+.000032+.000005$$

$$=.693148$$

(S=ln 2=.6931472 to 7D).