$.056 \pm .001$. From **3.8.1** the solution is

$$x=\tfrac{1}{2}(18.\underline{2}\pm[(18.\underline{2})^2-4(.05\underline{6})]^{\frac{1}{2}})$$
$$=\tfrac{1}{2}(18.\underline{2}\pm[33\underline{1.016}]^{\frac{1}{2}})=\tfrac{1}{2}(18.\underline{2}\pm 18.\underline{1939})$$
$$=18.\underline{1969},\ .00\underline{3}$$

The smaller root may be obtained more accurately from

* $$.05\underline{6}/18.\underline{1969}=.003\underline{1}\pm .0001.$$

Example 4. Compute $(-3+.0076i)^{\frac{1}{2}}$.

From **3.7.26,** $(-3+.0076i)^{\frac{1}{2}}=u+iv$ where

$$u=\frac{y}{2v},\ v=\left(\frac{r-x}{2}\right)^{\frac{1}{2}},\ r=(x^2+y^2)^{\frac{1}{2}}$$

Thus

$$r=[(-3)^2+(.0076)^2]^{\frac{1}{2}}=(9.00005776)^{\frac{1}{2}}=3.00000\ 9627$$

$$v=\left[\frac{3.00000\ 9627-(-3)}{2}\right]^{\frac{1}{2}}=1.73205\ 2196$$

$$u=\frac{y}{2v}=\frac{.0076}{2(1.73205\ 2196)}=.00219\ 392926$$

We note that the principal square root has been computed.

Example 5. Solve the cubic equation $x^3-18.1x-34.8=0$.

To use Newton's method we first form the table of $f(x)=x^3-18.1x-34.8$

x	$f(x)$
4	−43.2
5	− .3
6	72.6
7	181.5

We obtain by linear inverse interpolation:

$$x_0=5+\frac{0-(-.3)}{72.6-(-.3)}=5.004.$$

Using Newton's method, $f'(x)=3x^2-18.1$ we get

$$x_1\approx x_0-f(x_0)/f'(x_0)$$
$$\approx 5.004-\frac{(-.07215\ 9936)}{57.020048}\approx 5.00526.$$

Repetition yields $x_1=5.00526\ 5097$. Dividing $f(x)$ by $x-5.00526\ 5097$ gives $x^2+5.00526\ 5097x+6.95267\ 869$ the zeros of which are $-2.50263\ 2549\pm .83036\ 800i$.

Example 6. Solve the quartic equation

$$x^4-2.37752\ 4922x^3+6.07350\ 5741x^2-11.17938\ 023x+9.05265\ 5259=0.$$

Resolution Into Quadratic Factors
$(x^2+p_1x+q_1)(x^2+p_2x+q_2)$
by Inverse Interpolation

Starting with the trial value $q_1=1$ we compute successively

q_1	$q_2=\frac{a_0}{q_1}$	$p_1=\frac{a_1-a_3q_1}{q_2-q_1}$	$p_2=a_3-p_1$	$y(q_1)=q_1+q_2+p_1p_2-a_2$
1	9.053	−1.093	−1.284	5.383
2	4.526	−2.543	.165	.032
2.2	4.115	−3.106	.729	−2.023

We seek that value of q_1 for which $y(q_1)=0$. Inverse interpolation in $y(q_1)$ gives $y(q_1)\approx 0$ for $q_1\approx 2.003$. Then,

q_1	q_2	p_1	p_2	$y(q_1)$
2.003	4.520	−2.550	.172	.011

Inverse interpolation between $q_1=2.2$ and $q_1=2.003$ gives $q_1=2.0041$, and thus,

q_1	q_2	p_1	p_2	$y(q_1)$
2.0041	4.51706 7640	−2.55259 257	.17506 765	.00078 552
2.0042	4.51684 2260	−2.55282 851	.17530 358	.00001 655
2.0043	4.51661 6903	−2.55306 447	.17553 955	−.00075 263

Inverse interpolation gives $q_1=2.00420\ 2152$, and we get finally,

q_1	q_2	p_1	p_2	$y(q_1)$
2.00420 2152	4.51683 7410	−2.55283 358	.17530 8659	−.00000 0011

*See page II.