

Leibniz's Theorem for Differentiation of a Product**3.3.8**

$$\frac{d^n}{dx^n}(uv) = \frac{d^n u}{dx^n} v + \binom{n}{1} \frac{d^{n-1} u}{dx^{n-1}} \frac{dv}{dx} + \binom{n}{2} \frac{d^{n-2} u}{dx^{n-2}} \frac{d^2 v}{dx^2} + \dots + \binom{n}{r} \frac{d^{n-r} u}{dx^{n-r}} \frac{d^r v}{dx^r} + \dots + u \frac{d^n v}{dx^n}$$

3.3.9

$$\frac{dx}{dy} = 1 / \frac{dy}{dx}$$

3.3.10

$$\frac{d^2 x}{dy^2} = -\frac{d^2 y}{dx^2} \left(\frac{dy}{dx}\right)^{-3}$$

3.3.11

$$\frac{d^3 x}{dy^3} = -\left[\frac{d^3 y}{dx^3} \frac{dy}{dx} - 3 \left(\frac{d^2 y}{dx^2}\right)^2\right] \left(\frac{dy}{dx}\right)^{-5}$$

Integration by Parts**3.3.12**

$$\int u dv = uv - \int v du$$

3.3.13

$$\int uv dx = \left(\int u dx\right) v - \int \left(\int u dx\right) \frac{dv}{dx} dx$$

Integrals of Rational Algebraic Functions

(Integration constants are omitted)

3.3.14

$$\int (ax+b)^n dx = \frac{(ax+b)^{n+1}}{a(n+1)} \quad (n \neq -1)$$

3.3.15

$$\int \frac{dx}{ax+b} = \frac{1}{a} \ln |ax+b|$$

The following formulas are useful for evaluating $\int \frac{P(x)dx}{(ax^2+bx+c)^n}$ where $P(x)$ is a polynomial and $n > 1$ is an integer.

3.3.16

$$\int \frac{dx}{(ax^2+bx+c)} = \frac{2}{(4ac-b^2)^{\frac{1}{2}}} \arctan \frac{2ax+b}{(4ac-b^2)^{\frac{1}{2}}} \quad (b^2-4ac < 0)$$

3.3.17

$$= \frac{1}{(b^2-4ac)^{\frac{1}{2}}} \ln \left| \frac{2ax+b-(b^2-4ac)^{\frac{1}{2}}}{2ax+b+(b^2-4ac)^{\frac{1}{2}}} \right| \quad (b^2-4ac > 0)$$

3.3.18

$$= \frac{-2}{2ax+b} \quad (b^2-4ac=0)$$

3.3.19

$$\int \frac{xdx}{ax^2+bx+c} = \frac{1}{2a} \ln |ax^2+bx+c| - \frac{b}{2a} \int \frac{dx}{ax^2+bx+c}$$

3.3.20

$$\int \frac{dx}{(a+bx)(c+dx)} = \frac{1}{ad-bc} \ln \left| \frac{c+dx}{a+bx} \right| \quad (ad \neq bc)$$

3.3.21

$$\int \frac{dx}{a^2+b^2x^2} = \frac{1}{ab} \arctan \frac{bx}{a}$$

3.3.22

$$\int \frac{xdx}{a^2+b^2x^2} = \frac{1}{2b^2} \ln |a^2+b^2x^2|$$

3.3.23

$$\int \frac{dx}{a^2-b^2x^2} = \frac{1}{2ab} \ln \left| \frac{a+bx}{a-bx} \right|$$

3.3.24

$$\int \frac{dx}{(x^2+a^2)^2} = \frac{1}{2a^3} \arctan \frac{x}{a} + \frac{x}{2a^2(x^2+a^2)}$$

3.3.25

$$\int \frac{dx}{(x^2-a^2)^2} = \frac{-x}{2a^2(x^2-a^2)} + \frac{1}{4a^3} \ln \left| \frac{a+x}{a-x} \right|$$

Integrals of Irrational Algebraic Functions**3.3.26**

$$\int \frac{dx}{[(a+bx)(c+dx)]^{1/2}} = \frac{2}{(-bd)^{1/2}} \arctan \left[\frac{-d(a+bx)}{b(c+dx)} \right]^{1/2} \quad (bd < 0)$$

3.3.27

$$= \frac{-1}{(-bd)^{1/2}} \arcsin \left(\frac{2bdx+ad+bc}{bc-ad} \right) \quad (b > 0, d < 0)$$

3.3.28

$$= \frac{2}{(bd)^{1/2}} \ln |[bd(a+bx)]^{1/2} + b(c+dx)^{1/2}| \quad (bd > 0)$$

3.3.29

$$\int \frac{dx}{(a+bx)^{1/2}(c+dx)} = \frac{2}{[d(bc-ad)]^{1/2}} \arctan \left[\frac{d(a+bx)}{(bc-ad)} \right]^{1/2} \quad (d(ad-bc) < 0)$$

3.3.30

$$= \frac{1}{[d(ad-bc)]^{1/2}} \ln \left| \frac{d(a+bx)^{1/2} - [d(ad-bc)]^{1/2}}{d(a+bx)^{1/2} + [d(ad-bc)]^{1/2}} \right| \quad (d(ad-bc) > 0)$$