Errors in *The Elements of Nonlinear Optics* (1991) by Paul Butcher and David Cotter

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Below follows a listing of errors found in P. N. Butcher and D. Cotters book *The Elements of Nonlinear Optics* (Cambridge University Press, Cambridge, 1991). Being a summary of the notes I have made in my personal copy of the book since June 1996, this list should by no means be considered as any kind of "official" list of errors, but rather as an attempt to collect the (rather few) misprints in the text. In the list, not only typographical misprints, but also some inconsequent notations – which do not alter the described theory – are included.

p. 15 [*lines 14 and 16*] In order not to introduce any ambiguity of the multiple arguments of the symmetric and antisymmetric parts, the arguments $(t; \tau_1, \tau_2)$ should be explicitly written in the left-hand sides of the equations.

p. 15 [*line 18*] "... dummy variables $\alpha\tau_1$ and $\beta\tau_2$." should be replaced by "... dummy variables (α, τ_1) and (β, τ_2).", following the notation as used later in, for example, §2.3.2 and §4.3.1.

p. 49 [*line 31*] $H_1(t)$ should be replaced by $H_{\mathrm{I}}(t)$.

p. 54 [*lines 3, 4, and 6*] In Eq. (3.80), the upper limit of integration t_1 should be replaced by τ_1.

p. 54 [*lines 8 and 24*] In Eqs. (3.81) and (3.82), the upper limits of integration t_1 and t_{n-1} should be replaced by τ_1 and τ_{n-1}, respectively.

p. 60 [*line 24*] The sentence "To achieve this end we ..." should be replaced by "To achieve this we ..."

p. 66 [*line 11*] In the right hand side of Eq. (4.49), one should in order not to cause confusion with the Einstein convention of summation over repeated indices explicitly state that no summation is implied, and hence the equation should be written as

$$H_0 u_i(\Theta) = \mathbb{E}_i u_i(\Theta). \quad \text{(no sum)} \tag{4.49}$$

using the common notation as used in tensor calculus.

p. 67 [*line 6*] "... express the the unperturbed ..." should be replaced by "... express the unperturbed ..."

p. 72 [*line 15*] "$(\alpha, \omega_{1)}$" should be replaced by "(α, ω_1)".

p. 72 [*last line*] "... of this type, one of which is an identity." should be replaced by "... of this type, of which one is an identity."

p. 86 [*line 4*] In Eq. (4.103), "$\cdots \mathbf{f}_t(\omega_n \cdot \mathbf{e}_n\rangle$" should be replaced by "$\cdots \mathbf{f}_t(\omega_n) \cdot \mathbf{e}_n\rangle$".

p. 93 [*lines 12–13*] Strictly speaking, the real part of the susceptibility $\chi^{(1)}(-\omega_\sigma; \omega)$ is not proportional to the refractive index $n(\omega)$, but rather to $n^2(\omega) - 1$.

p. 97 [*lines 15 and 20*] Strictly speaking, Ω_{fg} is the transition angular frequency, and does not have the physical dimension of energy; therefore replace "Ω_{fg}" in lines 15 and 20 by "$\hbar\Omega_{fg}$".

p. 106 [*line 9*] In the first term of Eq. (4.128), the summation should be performed over index j rather than index i, i. e. replace $\sum_i \mathbf{p}_j$ by $\sum_j \mathbf{p}_j$.

p. 132 [*line 8*] In Eq. (5.30), "$E_i u_i(\Theta)$" should be replaced by "$\mathbb{E}_i u_i(\Theta)$ (no sum)".

p. 132 [*line 25*] "$\rho_0(a) = \eta \exp(-E_a/kT)$" should be replaced by "$\rho_0(a) = \eta \exp(-\mathbb{E}_a/kT)$".

p. 136 [*line 19*] In Eq. (5.43), "$\chi_{ua}(1)(-\omega,\omega)$" should be replaced by "$\chi^{(1)}_{ua}(-\omega,\omega)$".

p. 159 [*line 13*] In the left-hand side of Eq. (6.33), the Hamiltonian H_0 describing the system is a quantum-mechanical operator, while in the right-hand side, the matrix representation of the corresponding elements $\langle m|H_0|n\rangle = \delta_{mn}\mathbb{E}_n$ in the energy representation appears. In order to overcome this inconsistency, Eq. (6.33) should (in analogy with, for example, Eq. (6.31) for the matrix elements of the density operator) be replaced by either

$$\begin{pmatrix} \langle a|H_0|a\rangle & \langle a|H_0|b\rangle \\ \langle b|H_0|a\rangle & \langle b|H_0|b\rangle \end{pmatrix} = \begin{pmatrix} \mathbb{E}_a & 0 \\ 0 & \mathbb{E}_b \end{pmatrix},$$

or

$$\begin{pmatrix} [H_0]_{aa} & [H_0]_{ab} \\ [H_0]_{ba} & [H_0]_{bb} \end{pmatrix} = \begin{pmatrix} \mathbb{E}_a & 0 \\ 0 & \mathbb{E}_b \end{pmatrix}.$$

p. 160 [*line 2*] In the left-hand side of Eq. (6.34), the Hamiltonian $H_{\rm I}(t)$ is a quantum-mechanical operator, while in the right-hand side, the matrix representation of its scalar elements $\langle a|H_{\rm I}(t)|b\rangle$ appears. (The same inconsistency appear in Eq. (6.33).) In order to overcome this inconsistency, Eq. (6.34) should (in analogy with, for example, Eq. (6.31) for the matrix elements of the density operator) be replaced by either

$$\begin{pmatrix} \langle a|H_{\rm I}(t)|a\rangle & \langle a|H_{\rm I}(t)|b\rangle \\ \langle b|H_{\rm I}(t)|a\rangle & \langle b|H_{\rm I}(t)|b\rangle \end{pmatrix} = \begin{pmatrix} \delta\mathbb{E}_a & -e\mathbf{r}_{ab}\cdot\mathbf{E}(t) \\ -e\mathbf{r}_{ba}\cdot\mathbf{E}(t) & \delta\mathbb{E}_b \end{pmatrix},$$

or

$$\begin{pmatrix} [H_{\rm I}(t)]_{aa} & [H_{\rm I}(t)]_{ab} \\ [H_{\rm I}(t)]_{ba} & [H_{\rm I}(t)]_{bb} \end{pmatrix} = \begin{pmatrix} \delta\mathbb{E}_a & -e\mathbf{r}_{ab}\cdot\mathbf{E}(t) \\ -e\mathbf{r}_{ba}\cdot\mathbf{E}(t) & \delta\mathbb{E}_b \end{pmatrix}.$$

p. 164 [*line 15*] In Eq. (6.49), "$i\hbar(1-\rho_{bb})/T_b$" should be replaced by "$i\hbar(1-\rho_{aa})/T_b$".†

p. 203 [*lines 32–33*] "Fig. 4.3" should be replaced by "Fig. 4.4(a)".

p. 215 [*line 11*] In Eq. (7.14),

$$\ldots = \mu_0 \int_{-\infty}^{\infty} d\omega'(\omega+\omega')^2 \cdots$$

should be replaced by

$$\ldots = \frac{1}{c^2} \int_{-\infty}^{\infty} d\omega'(\omega+\omega')^2 \cdots$$

p. 220 [*section 7.2.1*] In the example of second harmonic generation, the wave equation (7.26) is given without any explanation of which point symmetry classes of media it applies to, and hence it is from the text virtually impossible to relate the effective nonlinear parameters to the elements of $\chi^{(2)}_{\mu\alpha\beta}(-\omega_\sigma;\omega,\omega)$.

p. 234 [*line 31*] In Eq. (7.45), "$\ldots = iq^*\hat{E}_3^* >$" should be replaced by "$\ldots = iq^*\hat{E}_3^*$".

p. 240 [*line 6*] In the first line of Eq. (7.55), there is an ambiguity of the denominator, as well as an erroneous dispersion term, and the equation

$$u = \tau\sqrt{n_2\omega/c|d^2k/d\omega^2|^2}\widehat{E}$$

should be replaced by

$$u = \tau\sqrt{n_2\omega/(c|d^2k/d\omega^2|)}\widehat{E}.$$

(The other lines of Eq. (7.55) are correct.)

† Cf. M. D. Levenson and S. S. Kano, *Introduction to Nonlinear Laser Spectroscopy* (Academic Press, New York, 1988), p. 33, Eqs. (2.3.1)–(2.3.2).

p. 241 [*line 30*] The fundamental bright soliton solution to the nonlinear Schrödinger equation should yield "$u(\zeta, s) = \mathrm{sech}(s)\exp(i\zeta/2)$", that is to say, *without* any minus sign in the exponential.

p. 251 [*line 6*] In Eq. (8.5), there is parenthesis mismatch in both right- and lefthand sides;

$$f_0[(\mathbb{E}_n(\mathbf{k})] = \{\exp[\mathbb{E}_n(\mathbf{k}) - \mathbb{E}_\mathrm{F}]/kT + 1\}^{-1}$$

should be replaced by

$$f_0[\mathbb{E}_n(\mathbf{k})] = \{\exp[(\mathbb{E}_n(\mathbf{k}) - \mathbb{E}_\mathrm{F})/kT] + 1\}^{-1}.$$

p. 252 [*line 6*] In Eq. (8.7), $n_\mathrm{ph}(\omega_s(\mathbf{q}))$ should be replaced by $n_\mathrm{ph}[\omega_s(\mathbf{q})]$, in order to follow the functional style of notation as used in, for example, Eq. (8.5).

p. 253 [*line 20*] In Eq. (8.11), insert a "]" after $\mathbb{E}_n(\mathbf{k})$.

p. 298 [*Table A3.2*] "... no centre of symmetry ..." should be replaced by "... no centre of inversion ...".

p. 317 [*lines 1, 9, and 24*] In Appendix 9, there is an inconsistency in the notation for the polarisation density and the electric dipole operator, as compared to the one used in Chapters 3 and 4. While $\mathbf{P}^\mathrm{D}$, $\mathbf{P}^\mathrm{Q}$, and $\mathbf{M}^\mathrm{D}$ in Eq. (A9.1) (and in line 9 on the same page) denote the all-classical electric dipolar, electric quadrupolar and magnetic dipolar polarization densities of the medium, they in Eqs. (A9.6) and (A9.7) clearly denote quantum-mechanical operators. In order to overcome this inconsistency in notation, which in addition gives a wrong answer if properly inserted into the perturbation calculus etc., one should chose either of the conventions. By choosing $\mathbf{P}^\mathrm{D}$, $\mathbf{P}^\mathrm{Q}$, and $\mathbf{M}^\mathrm{D}$ to denote the corresponding quantum-mechanical operators, which seem to be the easiest way of correcting this inconsistency, the following corrections to the text should be made:

[*line 1*]

$$\mathbf{P} = \mathbf{P}^\mathrm{D} + \mathbf{P}^\mathrm{Q}, \qquad \mathbf{M} = \mathbf{M}^\mathrm{D}, \tag{A9.1}$$

should be replaced by

$$\mathbf{P} = \langle\mathbf{P}^\mathrm{D}\rangle + \langle\mathbf{P}^\mathrm{Q}\rangle, \qquad \mathbf{M} = \langle\mathbf{M}^\mathrm{D}\rangle, \tag{A9.1}$$

[*line 9*] Remove "$\mathbf{P}^\mathrm{D}$" or replace with "$\langle\mathbf{P}^\mathrm{D}\rangle$".

[*line 24*] Somewhere in Appendix 9, there should be a clarifying statement that the nabla operator appearing in Eq. (A9.7) only is operating on the all-classical, macroscopic electric field of the light, and hence should be regarded as a classical vector when evaluating the quantum-mechanical trace that is involved in the expectation value of, for example, the corrected form of Eq. (A9.1).

p. 318 [*line 12*] "$\mathbf{M} << \mathbf{H}$" should be replaced by "$|\mathbf{M}| \ll |\mathbf{H}|$".

p. 318 [*line 23*] "$\mathbf{e}_j \cdot e\mathbf{r} + i\mathbf{k}\cdot\mathbf{q}\cdot\mathbf{e}_j + \mathbf{m}\cdot(\mathbf{k}\times\mathbf{e}_j)/\omega$" should be replaced by the same expression, though with each term divided by e.

p. 333 [*line 5*] In reference Manley, J. M. and Rowe, H. E. (1956), the page numbers should yield 904 − 14.

p. 334 [*line 44*] "Terhune, R. W. and Weinburger, D. A. ..." should be replaced by "Terhune, R. W. and Weinberger, D. A. ...".