Invited Paper

Journal of Nonlinear Optical Physics & Materials
Vol. 13, No. 1 (2004) 129–154

World Scientific
www.worldscientific.com

OPTICAL AMPLITUDE AND PHASE EVOLUTION IN NONLINEAR MAGNETO-OPTICAL BRAGG GRATINGS

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Received 30 September 2003

We present a theory on stratified nonlinear magneto-photonic crystals, with arbitrary spatial distribution of the refractive index and magneto-optical coefficients. The coupled mode theory is formulated for plane waves inside the crystal, and a perturbation series of integral solutions for the spectral response of the crystal is presented. Material issues and configurations are discussed, and the described theory is applied to the calculation of transmission spectra of binary magneto-photonic crystals.

Keywords: Nonlinear optics; magneto-optical devices; optical materials; optical elements, devices and systems.

1. Introduction and General Aspects

Here we address and assess the use of magneto-optic processes to perform certain functions, in particular as regards the manipulation of the polarization state of a beam, and the introduction of magneto-photonic structures and microcavities to enhance the weak magneto-optic strength that usually prevails there.

The study and use of magneto-optical effects did not follow the same pace as the nonlinear optical and electro-optical effects. In part this can be traced to the reduced strength of the magnetic dipolar coupling, with respect to that of the electric dipolar one, but also in part to the frequent overlook that the relevant features which are predominantly involved in the magneto-optical interactions are not the same as the ones in the case of all optical or electro-optical ones. Thus in contrast to the latter which predominantly affect the longitudinal characteristics of the beam propagation, in particular its wave vector and frequency, the former have their most striking impact on the polarization state of the beam, introducing outstanding sensitivity and robustness in the optical interactions. But most and

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foremost the applied static magnetic field in the case of magneto-optical interactions breaks the time-reversal symmetry and introduces a non-reciprocity in the beam interactions, with striking repercussions on the polarization state behavior which cannot be straightforwardly mimicked with all optical effects. It is precisely these aspects that constitute the distinctly new feature in the magneto-optical effects and where the effort should be directed.

Still the smallness of the magneto-optical coefficients, in particular in the transparency regime of the dielectrics, has precluded the consequent exploitation of these aspects and restricted their use in some applications that are not directly implemented in the all optical information storage, transmission, redistribution, manipulation that exclusively rely on nonlinear or electro-optical effects. There has been noticeable progress[2,1] in recent years in the conception of materials with enhanced magneto-optical coefficients, in particular Faraday and magneto-optic Kerr coefficients. In addition, in the case of nonlinear optical and electro-optical materials, whose performances were greatly improved through the electromagnetic and quantum confinement and the fabrication of photonic structures thereof,[3−8] recent studies show that the same can be exploited in the case of the magneto-optic materials through the development of magneto-photonic structures.

As in the case of the photonic gratings obtained by the spatial periodic modulation of the electric dipolar coupling, or equivalently of the dielectric constant, one can also simultaneously superimpose on such photonic modulation a spatial modulation of the magnetic dipolar coupling, or equivalently of the magnetization state over a photonic grating, and exploit the redistribution of the density of states of the electromagnetic field with the appearance of gaps and resonances that profoundly affect and selectively enhance the propagation of the electromagnetic waves and concomitantly the spectral features that the structure can sustain. But in contrast to the case of pure photonic structures where time-reversal symmetry strictly applies, leading to the propagation being reciprocal, in the case of the magneto-photonic ones the time-reversal symmetry breaks down and the non-reciprocity becomes an intrinsic feature of the interactions; this leads among other things to a distinction between forward and backward propagation in time and space which profoundly affects the polarization state of the beam since the interference patterns between the beams is drastically affected, as is also their coupling.

With the enhancement of these features through the photonic and magneto-photonic confinement and spatial modulation, the strength of the nonlinear effects with respect to light intensity also becomes relevant and in fact the photoinduced regime can drastically modify the linear regime; in the case of magneto-photonic gratings (MPGs) it can as much as drastically affect the degree of non-reciprocity and under extreme conditions even suppress the latter.

Here we review some aspects of the magneto-photonic gratings, single out the manifestation of the non-reciprocity in their characteristics, and subsequently assess the interplay of the non-reciprocity and the optical nonlinearity. In Sec. 2 we review the material aspects of the magneto-photonic gratings and the material

choices and values of magneto-optic coefficients that presently prevail in the ongoing fabrication effort. In Sec. 3 we recapitulate the nonlinear constitutive optical and magneto-optical relations, and discuss their reciprocity considerations in Sec. 4, and the coupled mode theory of operation of magneto-photonic structures in Sec. 5. In Secs. 6 and 7 we sketch the general procedures for obtaining the transmission and reflection characteristics of some specific magneto-photonic gratings, in particular the periodic and chirped ones, and we also discuss and compare with the case of inclusion of gyrotropic defects in photonic gratings, namely magneto-photonic microcavities, as an alternative. We then sketch the nonlinear or photoinduced regime in such structures and discuss some specific cases of photoinduced modification of the transmission and reflection characteristics of these magneto-photonic gratings. We conclude with some possible extensions and applications in devices.

2. Magneto-Photonic Gratings and Microcavities. Materials and Configurations

Recently,[3–8] a strong interest is emerging in the study of magneto-photonic gratings and microcavities and some noticeable progress has been achieved towards their fabrication. Besides the study of their intrinsic features, there is also the possibility of enhancing and improving the performances of several important magneto-optic devices such as polarization modulators, isolators and sensors that increasingly find their way into high performance laser systems and optical information technology although not yet into the main chain of optical information processing where at present only the photonic ones seem to be envisioned.

As with the case of photonic gratings one can envision a multitude of magneto-photonic gratings in particular with regard to their dimensionality. For the nonlinear magneto-optic regime we wish to address here, it suffices to consider one-dimensional (1D) magneto-photonic gratings and microcavities. It turns out that this is the configuration where the non-reciprocity also has its main impact (Faraday configuration). If we restrict ourselves in the case of one-dimensional gratings formed by piling up an equal number of identical magnetic and non-magnetic layers with the light beam incident normally on this stack we may still have several possible configurations differing by the order the piling is made, whether periodic or not, and with or without insertion of "gyrotropic defects"; the latter is a magnetic layer, formed from a ferro/ferri-magnetic material or a magnetic impurity-doped dielectric/semiconductor, with specific gyrotropic properties that are also frequency dependent and resonantly enhanced.

The main problem with the bulk magneto-optic materials is the difficulty to improve their gyrotropic properties, for instance their Faraday rotation strength, without a concomitant increase in their absorption; this is detrimental for their consideration in optical propagation applications or in extended and repeated solicitation of their gyrotropic function because of the cumulative effect on the optical signal intensity. With the recent progress in improving the gyrotropic properties of

dielectrics by doping with heavy magnetic impurity ions and the emerging progress in the fabrication of periodic structures thereof where magnetically doped and undoped layers of appropriate thickness are periodically stacked, a definite interest has been expressed in the study and exploitation of such structures and the new functionalities that can be introduced. An alternative approach is to use a quantum confined magnetic layer sandwiched between two photonic structures, namely a microcavity.[9] These two cases, namely the periodically distributed and the localized magneto-photonic coupling cases, will briefly be presented below and assessed for certain magneto-optic interactions and functions. The more general case of intermingling these two configurations and studying the resulting characteristics can also be discussed by extending the same procedures although with some considerable complication in the numerics and will not be attempted here.

In its most elementary configuration a 1D magneto-photonic grating is the singly periodic one formed by a repeat unit consisting of a pair of a magnetic and a nonmagnetic layer both assumed lossless and optically isotropic and in general of different thicknesses[3,5,7]; optically anisotropic layers can also be considered but this unnecessarily complicates the analysis and blurs the essential functions we wish to stress here and similarly for optically lossy ones. In practice however both cannot be avoided, in particular the optical losses, and provisions must then be made to assess their effect on the performances of the magneto-photonic structure. On the other hand one can straightforwardly extend the fabrication and analysis for non-periodic MPG where the magnetic and nonmagnetic layers are more or less randomly distributed, still keeping their respective total thicknesses and numbers the same as in the periodic case. One can also insert gyrotropic defects, a layer with distinct magneto-optic properties from those of the stacks and appropriately adapt their characteristics with those of the magneto-photonic structure; care must be paid here to avoid losses and destructive interferences.[5]

It goes without saying that in the singly periodic MPG there is also an underlying photonic grating of same period; the case of different periods for these two superimposed gratings could in principle be envisioned but its fabrication and potential usefulness is not evident. As in the case of the single periodic photonic grating the central role is played by the Bragg resonance condition with the essential difference now that this is polarization state dependent and different for the left and right circularly-polarized components of a linearly-polarized beam. To the extent that the Bragg resonance conditions determine the resonances in the transmission and reflection characteristics of the MPG this indicates that these are different for the left and right circularly-polarized modes or equivalently, the forward and backward propagating modes of same circular polarization experience different characteristics. This is a manifestation of the non-reciprocity and introduces the possibility of making photonic structures with direction-sensitive functions including spatial recognition and velocity sensitiveness.

The Bragg resonance characteristics besides their dependence on the layer thicknesses are also strongly conditioned by the refractive index contrast of the two layers

and this affects both their position and width; in the case of the magneto-photonic ones, because the degeneracy for left and right circular polarizations is lifted, each "photonic" Bragg resonance is split into two, corresponding to the two circularly-polarized components. The splitting between the two components is a measure of the circular birefringence or equivalently the Faraday rotation strength in the magnetic layer.

Clearly a refractive index change, linear or circular, can also be enforced by an intense light beam through photoinduced effects, the optical Kerr and the photoinduced Faraday effects respectively, and it is anticipated then that the Bragg resonance characteristics can be modified and to some extent controlled by photoinduced processes. Clearly for this to occur the total length of the photonic structure must be such that the cumulative effect results in a substantial global optical phase shift.[9]

The previous considerations can also be transposed to the case of a magneto-photonic microcavity.[8] Here a magnetic layer with thickness d_m is sandwiched between two identical periodic photonic gratings (actually the asymmetric case of two periodic photonic gratings with different repeat units can also be fabricated and treated without much additional complication and in fact has definite advantages over the symmetric one). The magnetic layer, also termed gyrotropic defect, is formed by a ferromagnetic metal, ferromagnetic material or magnetic impurity-doped dielectric such as semimagnetic semiconductor and most relies on the resonant enhancement of the gyrotropy; in the case of the metals and semiconductors the quantum confinement of the electrons can be exploited to substantially enhance the magneto-optic response of the layer and this is additionally enhanced by the confinement inside the microcavity formed from the two photonic gratings; the gyrotropic function springs from the Zeeman effect, the splitting of a transition by the applied magnetic field, which in essence introduces a doublet of gyrotropic defects one for each circular polarization. This configuration with the localized magneto-photonic defect has certain advantages over the previous ones, where the magneto-photonic coupling is periodically distributed, as far as the fabrication (for all constituent elements and their interface this is well advanced and controlled) and the analysis are concerned (the two photonic Bragg gratings being replaced by equivalent mirrors and the whole being treated as a Fabry–Pérot microcavity). However, it also has some serious disadvantages as it involves and solely relies on a gyrotropic material resonance with concomitant losses and energy deposition inside the microcavity.

Regarding the materials, in the case of the singly periodic magneto-photonic gratings the choice is presently focused on the bismuth doped yttrium iron garnets (Bi:YIG) for the magnetic layers and the gallium/gadolinium yttrium garnets (GG:YIG) or plain silicate glass (SiO_2) for the non-magnetic layer. Regarding the choice of the magnetic layer the figure of merit used is the ratio (specific Faraday rotation)/(absorption coefficient). The choice of the yttrium iron garnet matrices is made because of their very low absorption in the infrared spectral region (the

lowest among the ferrimagnetic dielectrics) the possibility to grow them in layers of excellent optical quality, the relatively low refractive index (typically $n_0 = 2.185$ or $\varepsilon_r = 4.75$) and well controlled magnetic domain structure; the bismuth possesses one of the largest known spin-orbit splitting strengths (this being a relativistic effect proportional to the ion mass and atom number Z) which leads to enhanced gyromagnetic factors and gyrotropic strengths and is now increasingly used in magnetic doping for magneto-optic isolators, modulators and sensors. Unfortunately too high Bi concentrations in the YIG may affect the domain structure, inducing instabilities there and also introducing substantial absorption losses and even optical anisotropy. In good quality Bi:YIG layers very weak magnetic field strengths are needed to orient the magnetic domains to saturation and reach Faraday rotations of a degree over a wavelength through the resulting spontaneous internal magnetic field. One has achieved very good performances in the interfacing of such magnetic Bi:YIG layers with non-magnetic layers, in particular with the case of gallium gadolinium yttrium iron garnets, with good lattice constant matching and appropriate optical properties for use in magneto-photonic gratings their refractive index being typically $n_r = 1.921$ or $\varepsilon_0 = 3.71$ and leading to an average refractive index contrast $\Delta n_0 = 0.264$ for the Bi:YIG/GG:YIG pair. To increase this contrast one may replace the GG:YIG with plain glass $n_0 = 1.50$ or $\varepsilon_r = 2.25$ and $\Delta n_0 = 0.685$ for the Bi:YIG/SiO_2 pair but the growth of such MPG still needs additional study and improvement. By proper choice of the grating characteristics one can operate the MPG either on transmission (Faraday) or reflection (magneto-optic Kerr) mode.

In the case of the magneto-photonic microcavities the most relevant studies have been performed with semimagnetic semiconductor microcavities,[8] consisting of a semimagnetic semiconductor quantum well sandwiched between two non-identical photonic Bragg mirrors, also fabricated with semimagnetic semiconductor layers for lattice matching purposes but with different magnetic impurity concentrations, so that the spectral features are different from those of the sandwiched quantum well; a static magnetic field is applied along the axis of the microcavity. The semimagnetic semiconductors are typically II–VI compounds, e.g. CdTe or CdSe, with the II-element partially replaced (substitutionally) with a transition element magnetic ion, e.g. Mn^{+2}; the cubic stucture and tetrahedral sp^3 bonding are preserved, but a spin exchange interaction between the band electrons and impurity d-electrons leads to a dramatic increase in the effective gyromagnetic (Landé) factor, typically two orders of magnitude at low temperatures (less than 12 K) but monotonically decreasing with higher temperatures. This leads to giant Zeeman splittings and concomitantly giant Faraday rotations. The molecular beam epitaxial growth technique allows perfect control of all the relevant optical features and doping here. Actually, by variable transverse doping one effectively obtains on the same structure a whole series of magneto-photonic microcavities with different characteristics: the Bragg resonance condition can be varied in the transverse direction, as can the spectral features of the quantum confined resonances in the quantum well.

Magneto-photonic microcavities were also obtained by sandwiching a ferromagnetic metal layer, ex cobalt, between two photonic Bragg gratings, and one can envision doing the same between two magneto-photonic Bragg gratings. Being ferromagnetic and possessing cubic symmetry, cobalt has large Faraday rotations but also large absorptions. For this reason, which is also valid for the semimagnetic semiconductor microcavities, one uses the magneto-optic Kerr effect configuration and operates in reflection mode.

We recall that for an isotropic magneto-optic medium in static magnetization state $\mathbf{M}_0$ the relation between the dielectric induction $\mathbf{D}_\omega$ and the electric field $\mathbf{E}_\omega$ of the propagating electromagnetic field (for which we adopt the convention $\exp(i\mathbf{k}\cdot\mathbf{r}-\omega t)$ for plane waves) in the Faraday configuration is given by the relation[10]

$$\mathbf{D}_\omega = \varepsilon_0(\varepsilon_\mathrm{r}\mathbf{E}_\omega + i\mathbf{E}_\omega \times \mathbf{g}), \tag{1}$$

where ε_r is the dielectric constant, a scalar in the present case of isotropic medium with $\varepsilon_\mathrm{r} = n^2$, and $\mathbf{g}$ is the the gyration vector with $\mathbf{g} = \mu_0\gamma\mathbf{M}_0$, where γ is the gyrotropic strength, and related to the specific Faraday rotation angle Φ_F (polarization rotation angle per unit length) and the Verdet constant V (rotation angle per unit path, per unit magnetic field strength) through the relation

$$\Phi_\mathrm{F} = \frac{\omega g}{2n_0 c} = VB_0\,,$$

as conforming to the classical/historical conventions of notation and units. We also find that the propagation eigenmodes are now the left and right circularly-polarized modes with refractive indices $n_\pm = n_0(1 \mp g/2n_0^2)$ and polarization states $\mathbf{e}_\pm = (\mathbf{e}_x \pm i\mathbf{e}_y)/\sqrt{2}$; this difference in refractive indices for left and right circularly-polarized light is also termed gyrotropy, and is measured with the g-parameter. The non-reciprocity in magneto-optics precisely springs from the vector product term in Eq. (1). Alternatively, and in line with the introduction of nonlinear susceptibility formalism as applied below, setting $\mathbf{D}_\omega = \varepsilon_0\mathbf{E}_\omega + \mathbf{P}_\omega$ and $\mathbf{P}_\omega = \mathbf{P}_\mathrm{e}^{(1)} + \mathbf{P}_\mathrm{m}^{(1)}$ where $\mathbf{P}_\mathrm{e}^{(1)} = \varepsilon_0\boldsymbol{\chi}^{(\mathrm{ee})}\mathbf{E}$, one may describe the vector product in Eq. (1) as a magnetization induced polarization term and write

$$\mathbf{P}_\mathrm{m}^{(1)} = \varepsilon_0\boldsymbol{\chi}^{(\mathrm{eem})}\mathbf{E}_\omega\mathbf{B}_0\,, \tag{2}$$

where $\mathbf{B}_0 = \mu_0(\mathbf{H}_0 + \mathbf{M}_0)$ and in the case of a magnetic medium only the magnetization matters. In Eq. (2), $\boldsymbol{\chi}^{(\mathrm{eem})}$ is the second-order magneto-optical susceptibility tensor, of rank three. For isotropic media, one easily finds that the relation (2) can be written as the cross product appearing in Eq. (1), with

$$\mathbf{g} = -i\chi_{xyz}^{(\mathrm{eem})}\mathbf{B}_0$$

being an axial vector, or pseudo-vector.[11] One may alternatively write this as an effective first-order rank-two susceptibility[1,2] $\boldsymbol{\chi}^{(1)}(\mathbf{M}_0) = \boldsymbol{\chi}^{(\mathrm{ee})} + \boldsymbol{\chi}^{(\mathrm{eem})}\mathbf{B}_0$. These coefficients satisfy the Onsager relations that are another manifestation of the non-reciprocity brought in by the static magnetic field. Actually the previous relations can also be derived starting from the appropriate thermodynamic potential.

For intense optical fields the material coefficients $\varepsilon_{\rm r}$ and $\mathbf{g}$ undergo photo-induced changes which can be described in terms of nonlinear susceptibilities; for the nonlinear regime of the photonic and magneto-photonic gratings we are considering here (with respect to light intensity) the lowest order ones are the photo-induced refractive index change, or optical Kerr effect, and the photo-induced Faraday effect, respectively related to the polarization terms

$$\mathbf{P}_{\rm e}^{(3)} = \varepsilon_0 \boldsymbol{\chi}^{(\rm eeee)} \mathbf{E}_\omega \mathbf{E}_\omega \mathbf{E}_\omega^* \,,$$
$$\mathbf{P}_{\rm m}^{(3)} = \varepsilon_0 \boldsymbol{\chi}^{(\rm eeeem)} \mathbf{E}_\omega \mathbf{E}_\omega \mathbf{E}_\omega^* \mathbf{B}_0 \,.$$

We have assumed that the medium has inversion symmetry and we only concentrate on the response at a single frequency ω. One may alternatively write this as an effective third-order polarization density

$$\mathbf{P}^{(3)} = \mathbf{P}_{\rm e}^{(3)} + \mathbf{P}_{\rm m}^{(3)} = \varepsilon_0 \boldsymbol{\chi}^{(3)}(\mathbf{M}) \mathbf{E}_\omega \mathbf{E}_\omega \mathbf{E}_\omega^* \,,$$

where $\boldsymbol{\chi}^{(3)}(\mathbf{M}) = \boldsymbol{\chi}^{(\rm eeee)} + \boldsymbol{\chi}^{(\rm eeeem)} \mathbf{B}_0$ is the rank-four effective third-order susceptibility tensor. These coefficients also satisfy Onsager type relations. While the complex character of the $\boldsymbol{\chi}^{(n)}$ only reflects the non-reciprocity when the partial $\boldsymbol{\chi}$'s are real (no absorption losses), when absorption losses are present the partial $\boldsymbol{\chi}$'s are complex and one must pay some attention in the formulation of the Onsager relations. One can derive quantum mechanical microscopic expressions for the magneto-optic coefficients using the same perturbation techniques as for the optical ones, and show that all the macroscopic symmetry properties are incorporated there too including the non-reciprocity and the ones resulting from spatial symmetry operations (e.g. crystalline symmetry).

In general it is very hazardous to roughly estimate the magneto-optic coefficients, much more so to evaluate them starting from their quantum mechanical expressions; it is here important to remind ourselves that magnetism is a quintessentially quantal and a relativistic manifestation at a macroscopic scale. The reasons for the computational complexity are diverse and well identified: the computation of magnetic dipole transition moments has several contributions, spin and orbital, the latter involving the electron impulse operator which, in contrast to electric dipole operator, is difficult to calculate, etc. The situation of course is increasingly complicated as we move up into higher-order magneto-optic susceptibilities. We may get a rough relative estimate however by using some simple scaling arguments. On inspection of the quantum expressions thus one may roughly set

$$\boldsymbol{\chi}^{(\rm eeeem)} \mathbf{B}_0 / \boldsymbol{\chi}^{(\rm eeee)} \approx \boldsymbol{\chi}^{(\rm eem)} \mathbf{B}_0 / \boldsymbol{\chi}^{(\rm ee)} = g / \chi_{xx}^{(\rm ee)},$$

and from the definition of the Faraday rotation angle in terms of the Verdet constant one hence gets

$$\boldsymbol{\chi}^{(\rm eeeem)} \mathbf{B}_0 / \boldsymbol{\chi}^{(\rm eeee)} \approx \frac{2ncVB_0}{\omega \chi_{xx}^{(\rm ee)}} = \frac{\lambda_0 n \Phi_{\rm F}}{\pi(n^2-1)} = \kappa \,.$$

Thus for glass (SiO_2) with a Verdet constant of $V = 10$ rad/Tm one gets $\kappa = 0.4 \times 10^{-5}$ for $B_0 = 1$ T, while for Bi:YIG with $\Phi_F = 0.5$ deg/μm one gets close to $\kappa = 0.2 \times 10^{-2}$, a very comfortable number for applications. The above arguments are valid away from resonances, in the transparency region of the material, where the absorption losses are minimal and hence appropriate for magneto-photonic gratings either in reflection or transmission modes.

Close to resonances for magnetic layers to be inserted as gyrotropic defects in magneto-photonic gratings, for instance ferromagnetic metal layers or semimagnetic semiconductor quantum wells, one can actually evaluate or measure the relevant oscillator and gyration strengths quite accurately and compute the relevant coefficients; here the coefficients are resonantly enhanced and can reach very high values, with the effective Verdet constants becoming orders of magnitude higher with respect to the previous cases. Unfortunately close to a resonance absorption losses are also present and must also be considered and the figure of merit becomes then unfavorable for their use as magnetic layers in periodic magneto-photonic gratings. Close to resonances the dynamics become relevant because of the relaxation processes involved. This can be included using the Bloch equations for a pair of two-level systems, one for each polarization state since the magnetic field effectively duplicates a two-level system, the simplest model for a resonance, through the Zeeman effect. The predominant nonlinear mechanism here is the population saturation but in certain cases the magnetization also may be modified through photoinduced realignment of the magnetic moments, for instance through spin-orbit coupling. This introduces a whole new class of effects and potential applications that are not considered here.

3. The Induced Magneto-Optical Polarization Density

So far, the discussion of optical and magneto-optical interactions have been performed without entering the explicit form of the involved susceptibilities and polarization densities. We will now, from a fundamental starting point, describe in more detail the formalism underlying the coupled mode theory and further wave propagation aspects of nonlinear magneto-photonic devices.

As the natural harmonic temporal oscillation is separated from the real-valued observable fields, by introducing the field envelopes of the electric and magnetic fields as

$$\mathbf{E}(\mathbf{r}, t) = \mathrm{Re}[\mathbf{E}_\omega \exp(-i\omega t)]\,, \tag{3a}$$

$$\mathbf{B}(\mathbf{r}, t) = \mathrm{Re}[\mathbf{B}_\omega \exp(-i\omega t)]\,, \tag{3b}$$

respectively, and similarly for the electric and magnetic polarization densities $\mathbf{P}(\mathbf{r}, t)$ and $\mathbf{M}(\mathbf{r}, t)$, Maxwell's equations for wave propagation in a nonlinear magneto-optical Kerr-medium can be reformulated to yield the wave equation

$$\nabla \times \nabla \times \mathbf{E}_\omega - (\omega/c)^2 \mathbf{E}_\omega = \mu_0 \omega^2 \mathbf{P}_\omega\,, \tag{4a}$$

$$\nabla \times \nabla \times \mathbf{B}_\omega - (\omega/c)^2 \mathbf{B}_\omega = -i\mu_0 \omega \nabla \times \mathbf{P}_\omega\,. \tag{4b}$$

In Eqs. (4), the electric polarization density $\mathbf{P}_\omega = \mathbf{P}_\omega^{(\mathrm{o})} + \mathbf{P}_\omega^{(\mathrm{mo})}$ comprises the all-optical, electric dipolar contribution[12,13]

$$P_\omega^{(\mathrm{o})\mu} = \varepsilon_0[\chi_{\mu\alpha}^{(\mathrm{ee})} E_\omega^\alpha + \chi_{\mu\alpha\beta\gamma}^{(\mathrm{eeee})} E_\omega^\alpha E_\omega^\beta E_\omega^{\gamma *}] \,, \tag{5}$$

with optical susceptibilities[12]

$$\chi_{\mu\alpha}^{(\mathrm{ee})} = \chi_{\mu\alpha}^{(\mathrm{ee})}(-\omega;\omega) \,,$$
$$\chi_{\mu\alpha\beta\gamma}^{(\mathrm{eeee})} = \chi_{\mu\alpha\beta\gamma}^{(\mathrm{eeee})}(-\omega;\omega,\omega,-\omega) \,,$$

and the magneto-optical contribution

$$P_\omega^{(\mathrm{mo})\mu} = \varepsilon_0[\chi_{\mu\alpha\beta}^{(\mathrm{eem})} E_\omega^\alpha B_0^\beta + \chi_{\mu\alpha\beta\gamma\delta}^{(\mathrm{eeeem})} E_\omega^\alpha E_\omega^\beta E_\omega^{\gamma *} B_0^\delta] \,, \tag{6}$$

with magneto-optical susceptibilities[14,15]

$$\chi_{\mu\alpha\beta}^{(\mathrm{eem})} = \chi_{\mu\alpha\beta}^{(\mathrm{eem})}(-\omega;\omega,0) \,,$$
$$\chi_{\mu\alpha\beta\gamma\delta}^{(\mathrm{eeeem})} = \chi_{\mu\alpha\beta\gamma\delta}^{(\mathrm{eeeem})}(-\omega;\omega,\omega,-\omega,0) \,.$$

These tensor expressions for the polarization density, which apply to an arbitrary point symmetry group of the medium, are to be evaluated under the Einstein convention of summation over repeated indices of Cartesian coordinates.

For locally isotropic media, as considered here, the optical properties of any point of the medium are left invariant under any rotation or inversion in said point, keeping the geometry of all externally present fields fixed in space, and the tensor formulation (5) can be cast into the vectorial form

$$\begin{aligned}\mathbf{P}_\omega^{(\mathrm{o})} = \varepsilon_0[&\chi_{xx}^{(\mathrm{ee})} \mathbf{E}_\omega + \chi_{xyyx}^{(\mathrm{eeee})} (\mathbf{E}_\omega \cdot \mathbf{E}_\omega) \mathbf{E}_\omega^* \\ &+ (\chi_{xxxx}^{(\mathrm{eeee})} - \chi_{xyyx}^{(\mathrm{eeee})})(\mathbf{E}_\omega \cdot \mathbf{E}_\omega^*)\mathbf{E}_\omega] \,.\end{aligned}$$

This polarization density governs the linear electric displacement and optical Kerr-effect.[12] Similarly, the magneto-optical contribution can for isotropic media be cast into the vectorial form

$$\begin{aligned}\mathbf{P}_\omega^{(\mathrm{mo})} = \varepsilon_0[&\chi_{xyz}^{(\mathrm{eem})} \mathbf{E}_\omega \times \mathbf{B}_0 + \chi_{xyyyz}^{(\mathrm{eeeem})} (\mathbf{E}_\omega \cdot \mathbf{E}_\omega^*)(\mathbf{E}_\omega \times \mathbf{B}_0) \\ &+ \chi_{xxxyz}^{(\mathrm{eeeem})} \mathbf{E}_\omega (\mathbf{E}_\omega \cdot (\mathbf{E}_\omega^* \times \mathbf{B}_0)) \\ &+ \chi_{xyyzy}^{(\mathrm{eeeem})} (\mathbf{E}_\omega \times \mathbf{E}_\omega^*)(\mathbf{E}_\omega \cdot \mathbf{B}_0)] \,,\end{aligned}$$

governing the linear Faraday rotation[10] and photoinduced Faraday effect.[14–16] The vectorial forms (5) and (6) of the constitutive relations were derived using intrinsic permutation symmetry, which always is applicable to nonlinear susceptibilities. In the magneto-optical part of the polarization density, we neglected all terms which are nonlinear in the static magnetic field, hence omitting the Cotton–Mouton effect.[10] This approximation is justified whenever the static magnetic field is directed mainly in the direction of propagation of the light. If the Cotton–Mouton effect were to be included in the analysis, the terms

$$\chi^{(\mathrm{eemm})}_{xxyy}(\mathbf{B}_0 \cdot \mathbf{B}_0)\mathbf{E}_\omega + (\chi^{(\mathrm{eemm})}_{xxxx} - \chi^{(\mathrm{eemm})}_{xxyy})(\mathbf{E}_\omega \cdot \mathbf{B}_0)\mathbf{B}_0$$

should be added to $\mathbf{P}^{(\mathrm{mo})}_\omega$.

It should be noticed that the isotropy of the medium, which is related to point-symmetry properties under rotation and inversion, is distinct from any homogenity or inhomogenity of the medium, as being rather a translational property.

The linear terms in the electric polarization density are now collected into

$$\mathbf{P}^{(\mathrm{L})}_\omega = \varepsilon_0[(n^2 - 1)\mathbf{E}_\omega + i\mathbf{E}_\omega \times \mathbf{g}]$$

where $n = (1 + \chi^{(\mathrm{ee})}_{xx})^{1/2}$ and $\mathbf{g} = -i\chi^{(\mathrm{eem})}_{xyz}\mathbf{B}_0$ are the refractive index and the gyration vector of the medium, both being quantities that spatially vary in space. Similarly, the nonlinear terms in the polarization density are collected into

$$\begin{aligned}\mathbf{P}^{(\mathrm{NL})}_\omega = \varepsilon_0[&a_1(\mathbf{E}_\omega \cdot \mathbf{E}_\omega)\mathbf{E}^*_\omega + a_2(\mathbf{E}_\omega \cdot \mathbf{E}^*_\omega)\mathbf{E}_\omega \\ &+ ib_1(\mathbf{E}_\omega \cdot \mathbf{E}^*_\omega)(\mathbf{E}_\omega \times \mathbf{g}) + ib_2\mathbf{E}_\omega(\mathbf{E}_\omega \cdot (\mathbf{E}^*_\omega \times \mathbf{g})) \\ &+ ib_3(\mathbf{E}_\omega \times \mathbf{E}^*_\omega)(\mathbf{E}_\omega \cdot \mathbf{g})]\,,\end{aligned}$$

where we, for the sake of simplicity, eliminated the static magnetic field $\mathbf{B}_0$ in favor of the gyration vector $\mathbf{g}$. In an optically non-resonant regime, the nonlinear coefficients a_k and b_k are all real-valued scalars.

The optical Kerr-effect and the photo-induced Faraday effect have the same power dependence of the optical field, and in the Faraday configuration, with $\mathbf{g}$ directed along the direction of propagation of the light, they both affect the rotation of the polarization ellipse as the light traverses the medium. In particular, for elliptically-polarized light, the optical Kerr-effect accounts for the so-called ellipse rotation[17] of strong light beams. However, the symmetry properties of the optical Kerr-effect and the photo-induced Faraday effect are distinctly different: the optical Kerr-effect causes a change in the refractive index that is symmetrical with respect to the ellipticity of the light, that is to say, with zero rotary power for linearly polarized light; in contrast, the photoinduced Faraday effect essentially causes an anti-symmetric change in the refractive index, where one of the circular polarizations experiences a photo-induced increase in the refractive index, while the orthogonal polarization simultaneously experiences a decrease.

4. Reciprocity Considerations

A well-known fact in electrodynamic field theory is that the electric field $\mathbf{E}(\mathbf{r}, t)$, the electric polarization density $\mathbf{P}(\mathbf{r}, t)$, and the electric displacement $\mathbf{D}(\mathbf{r}, t)$ are all quantities that are left invariant under time reversal. This follows directly from the experimental fact that the electric charge density is a true scalar, which is left invariant under space inversion as well as time reversal.[11] As a consequence, from Faraday's law

$$\nabla \times \mathbf{E}(\mathbf{r}, t) = -\frac{\partial \mathbf{B}(\mathbf{r}, t)}{\partial t}\,,$$

and Maxwell–Ampère's law,

$$\nabla \times \mathbf{H}(\mathbf{r},t) = \frac{\partial \mathbf{D}(\mathbf{r},t)}{\partial t} + \mathbf{J}(\mathbf{r},t)\,,$$

the magnetic induction $\mathbf{B}(\mathbf{r},t)$, the magnetization $\mathbf{M}(\mathbf{r},t)$, and the magnetic field $\mathbf{H}(\mathbf{r},t)$ must all be pseudo-vectors, being odd quantities under time reversal. Taken all together, Maxwell's equations are, in their classical form, always left invariant under time reversal, regardless of any constitutive relations $\mathbf{D}[\mathbf{E},\mathbf{P}]$ or $\mathbf{H}[\mathbf{B},\mathbf{M}]$.

Since "non-reciprocity" is a term that is often used to indicate the breaking of invariance under time reversal of the wave propagation, considering the fact that Maxwell's equations are always left invariant under time reversal, this arises the question on what we really mean by non-reciprocal wave propagation.

In connection with time reversal, the beam reversal properties of a setup are often used to illustrate non-reciprocal effects, of which the linear Faraday effect probably is the most commonly-used example. This illustration is essentially based on the observation that, in the time-harmonic separation of Eqs. (3), the time reversal $t \to -t$ corresponds to an equivalent wave propagation picture with $(\mathbf{E}_\omega, \mathbf{B}_\omega) \to (\mathbf{E}^*_\omega, \mathbf{B}^*_\omega)$, that is to say, with the direction of propagation everywhere reversed. However, this image of beam reversal only gives the wave propagation picture, while the interpretation of time reversal properties should be applicable to any point of the medium of interest. Therefore, we will now recapitulate what we actually mean by non-reciprocal polarization effects.

From Eq. (5), it is clear that the all-optical constitutive relation is always left invariant under time reversal, since no pseudo-vectors, such as the magnetic induction of magnetization, are present. In the magneto-optical constitutive relation given in Eq. (6), however, an even number of pseudo-vectors (the magnetic induction) occur in each term of the right-hand side, while the left-hand side (a term of the electric polarization density) is a proper vector.

Since this seems to indicate the obvious contradiction that the right-hand side of Eq. (6) should change sign under time reversal, while the left-hand side should be left invariant under the same operation, this is one possible interpretation of the breaking of time reversal symmetry, or a signature of non-reciprocity. The clue to this obvious lack of consistency lies in the observation that the susceptibilities themselves are not necessarily real tensors, and that the optical and magneto-optical tensors have different transformation properties under time reversal.

The optical and magneto-optical susceptibilities are generally given in terms of the molecular electric and magnetic dipole moments as[12,14]

$$\chi^{(\mathrm{ee})}_{\mu\alpha}(-\omega_\sigma;\omega) \sim p^\mu_{ab} p^\alpha_{ba} / D_1(\omega)\,, \tag{7a}$$

$$\chi^{(\mathrm{eem})}_{\mu\alpha\beta}(-\omega_\sigma;\omega_1,\omega_2) \sim p^\mu_{ab} p^\alpha_{bc} m^\beta_{ca} / D_2(\omega_1,\omega_2)\,, \tag{7b}$$

$$\chi^{(\mathrm{eeee})}_{\mu\alpha\beta\gamma}(-\omega_\sigma;\omega_1,\omega_2,\omega_3) \sim p^\mu_{ab} p^\alpha_{bc} p^\beta_{cd} p^\gamma_{da} / D_3(\omega_1,\omega_2,\omega_3)\,, \tag{7c}$$

$$\chi^{(\mathrm{eeeem})}_{\mu\alpha\beta\gamma\delta}(-\omega_\sigma;\omega_1,\omega_2,\omega_3,\omega_4) \sim p^\mu_{ab} p^\alpha_{bc} p^\beta_{cd} p^\gamma_{de} m^\delta_{ea} / D_4(\omega_1,\omega_2,\omega_3,\omega_4)\,, \tag{7d}$$

where $p^{\mu}_{ab} = \langle a|\mathbf{e}_{\mu}\cdot\hat{\mathbf{p}}|b\rangle$ and $m^{\mu}_{ab} = \langle a|\mathbf{e}_{\mu}\cdot\hat{\mathbf{m}}|b\rangle$ are the matrix elements of the molecular electric and magnetic dipolar operators. In Eqs. (7), $D_k(\omega_1,\ldots,\omega_k)$ denote k-order polynomials in the angular frequency arguments $\omega_1,\ldots,\omega_k$, also involving the molecular transition frequencies Ω_{ab} and dephasing parameters $\pm i\Gamma_{ab}$.[12]

In the expressions (7), the matrix elements p^{μ}_{ab} are components of proper vectors (electric dipole transitions), while m^{μ}_{ab} are components of pseudo-vectors (magnetic dipole transitions). Since any odd-power dependence of the magnetic induction in Eqs. (6) occur together with an odd-power dependence of the magnetic dipole elements that appear in the susceptibilities, this means that, in fact, the invariance under time reversal still holds even for the magneto-optically induced terms of the polarization density. However, in the constitutive form (6), with the inherent magnetic dipole transitions concealed in the susceptibilities, the magneto-optical polarization clearly states the breaking of time-reversal, unless we consider in more detail the time reversal properties of the medium, and not only the dependence of the involved fields. This formulation of the overall invariance of the involved physical processes under time reversal originally stems from that the fact that the interaction Hamiltonian, taking electric and magnetic dipolar interaction into account, yields $H_{\mathrm{I}} = -\hat{\mathbf{p}}\cdot\mathbf{E}(\mathbf{r},t) - \hat{\mathbf{m}}\cdot\mathbf{B}(\mathbf{r},t)$. The invariance of the interaction Hamiltonian under time reversal is a basic foundation of the Onsager reciprocity relations.[18] This is what we imply by the term "non-reciprocity", with an apparant breaking of time reversal symmetry due to the form of the constitutive relations, unless we investigate in more detail the explicit form of the susceptibilities.

In this context, we should also notice that in the Cotton–Mouton effect, quadratic terms $(\mathbf{B}_0\cdot\mathbf{B}_0)\mathbf{E}_{\omega}$ and $(\mathbf{B}_0\cdot\mathbf{E}_{\omega})\mathbf{B}_0$ are part of the polarization density. Since this involves even products of the magnetic induction, this is still a strictly reciprocal effect within the above described mode of terminology.

The non-reciprocity of the medium has significant impact on the beam reversal and wave propagation properties of the device. It should be emphasized that the beam reversal properties, which essentially are given by a spatial inversion, should not be confused with the time reversal properties.

5. Coupled Mode Theory

The theory described so far is generally applied to the total electric field distribution inside an arbitrary magneto-photonic crystal. In order to concentrate on the filtering aspects, we will now, for simplicity, assume that the wave propagation is along a direction parallel to the linear direction of the externally applied static magnetic field $\mathbf{B}_0 = B_0\mathbf{e}_z$. We will also assume the infinite plane wave approximation to hold, giving the one-dimensional form of the wave equation (4a) as

$$\frac{\partial^2\mathbf{E}_{\omega}}{\partial z^2} + \frac{\omega^2}{c^2}[n^2(z)\mathbf{E}_{\omega} + i\mathbf{E}_{\omega}\times\mathbf{g}(z)] = -\mu_0\omega^2\mathbf{P}^{(\mathrm{NL})}_{\omega}\,. \tag{8}$$

In the case of a stratified medium, as considered here, with all material parameters being functions of the longitudinal coordinate z only, the electric dipolar refractive

index $n(z)$ of the medium is assumed to be modulated around a median value of n_0 and median period $\Lambda_{\rm ed}$, as

$$\frac{[n^2(z)-n_0^2]}{n_0} = a_{\rm ed}(z){\rm Re}[\exp(iK_{\rm ed}z + i\vartheta_{\rm ed}(z))]\,,$$

where $K_{\rm ed} = 2\pi/\Lambda_{\rm ed}$ is the magnitude of the median grating vector related to electric dipolar contributions. For a grating of spatially invariant modulation depth $a_{\rm ed} = {\rm const.}$, the local period of the refractive index profile is from this representation given in terms of the spatially varying phase function $\vartheta_{\rm ed}(z)$ as

$$\Lambda_{\rm ed}^{\rm (loc)} = \frac{\Lambda_{\rm ed}}{\left[1 + \dfrac{\Lambda_{\rm ed}}{2\pi}\dfrac{\partial\vartheta_{\rm ed}(z)}{\partial z}\right]}\,.$$

Similar to the electric dipolar contributions to the refractive index profile, the magnetic-dipolar, or magneto-optical, contributions are taken according to the representation

$$\frac{[g(z)-g_0]}{n_0} = a_{\rm md}(z){\rm Re}[\exp(iK_{\rm md}z + i\vartheta_{\rm md}(z))]\,,$$

where $g(z) = \mathbf{e}_z \cdot \mathbf{g}(z)$ is the projected gyration vector, and $K_{\rm md} = 2\pi/\Lambda_{\rm md}$ is the magnitude of the grating vector related to magnetic dipolar contributions. In the case with a homogeneous linear magneto-optic effect across the structure, $a_{\rm md}(z)$ is identically zero.

The optical field is separated into forward and backward propagating components as

$$\begin{aligned}\mathbf{E}_\omega = {}& \mathbf{e}_+ A_+^{\rm f}(z)\exp[i(\omega n_0/c)(1-g_0/2n_0^2)z] \\ & + \mathbf{e}_- A_-^{\rm f}(z)\exp[i(\omega n_0/c)(1+g_0/2n_0^2)z] \\ & + \mathbf{e}_+^* A_+^{\rm b}(z)\exp[-i(\omega n_0/c)(1+g_0/2n_0^2)z] \\ & + \mathbf{e}_-^* A_-^{\rm b}(z)\exp[-i(\omega n_0/c)(1-g_0/2n_0^2)z]\,,\end{aligned}$$

where $A_\pm^{\rm f}$ and $A_\pm^{\rm b}$ are the left/right circularly-polarized envelopes of the forward and backward propagating field components, expressed in the circularly-polarized basis vectors

$$\mathbf{e}_+ = (\mathbf{e}_x + i\mathbf{e}_y)\sqrt{2}, \qquad \mathbf{e}_- = (\mathbf{e}_x - i\mathbf{e}_y)\sqrt{2}\,.$$

Whenever backward and forward propagating waves interact, either through direct interaction through a local coupling due to material effects, or due to backscattering or reflection, non-reciprocity enters explicitly. In the following, we will denote the direction of propagation of light as the longitudinal direction, along the z-axis or a Cartesian coordinate system, while the x- and y-directions are denoted as the transverse direction. We will also assume the grating structure to be resonant mainly in reflection, with the modulation period of linear optical properties close to twice the magnitude of the wave vector of the propagating light.

By applying the slowly varying envelope approximation, separating orthogonal circular polarizations, and subsequently separating phase mismatched terms, the wave equation (8) becomes

$$\frac{\partial A_{\pm}^{\mathrm{f}}}{\partial z} - [\kappa_{\mathrm{ed}}^{*}(z)\exp(-i\beta_{\mathrm{ed}}^{\pm}z) \mp \kappa_{\mathrm{md}}^{*}(z)\exp(-i\beta_{\mathrm{md}}^{\pm}z)]A_{\mp}^{\mathrm{b}}$$
$$= -\mu_0\omega^2 P_{\omega\pm}^{(\mathrm{NL})}\exp[-i(\omega n_0/c)(1 \mp g_0/2n_0^2)z] \tag{9a}$$

$$\frac{\partial A_{\mp}^{\mathrm{b}}}{\partial z} - [\kappa_{\mathrm{ed}}(z)\exp(i\beta_{\mathrm{ed}}^{\pm}z) \mp \kappa_{\mathrm{md}}(z)\exp(i\beta_{\mathrm{md}}^{\pm}z)]A_{\pm}^{\mathrm{f}}$$
$$= \mu_0\omega^2 P_{\omega\pm}^{(\mathrm{NL})}\exp[i(\omega n_0/c)(1 \mp g_0/2n_0^2)z] \tag{9b}$$

where $P_{\omega\pm}^{(\mathrm{NL})} = \mathbf{e}_{\pm}^{*}\cdot\mathbf{P}_{\omega}^{(\mathrm{NL})}$, and where multiple scales analysis[19–21] was applied. In Eqs. (9) we defined the optical and magneto-optical coupling coefficients

$$\kappa_{\mathrm{ed}}(z) = -i(\omega/4c)a_{\mathrm{ed}}(z)\exp[-i\vartheta_{\mathrm{ed}}(z)]\,,$$
$$\kappa_{\mathrm{md}}(z) = -i(\omega/4c)a_{\mathrm{md}}(z)\exp[-i\vartheta_{\mathrm{md}}(z)]\,,$$

and the corresponding optical and magneto-optical detuning parameters

$$\beta_{\mathrm{ed}}^{\pm} = (2\omega n_0/c)(1 \mp g_0/2n_0^2) - K_{\mathrm{ed}}\,,$$
$$\beta_{\mathrm{md}}^{\pm} = (2\omega n_0/c)(1 \mp g_0/2n_0^2) - K_{\mathrm{md}}\,.$$

The coupled mode equations (9) were derived under the assumption that the light scattering due to the modulated linear refractive index and the gyration constant dominate over any amplitude change due to nonlinear effects. These coupled mode equations are analogous to those of linear all-optical Bragg gratings,[22,23] though with the difference that the effective coupling constant as presented here involves magnetic dipolar interactions as well, splitting the degeneracy between left and right circularly-polarized modes.

6. Direct Scattering Approach. Perturbation Analysis

Taking into account the explicit form of the nonlinear polarization density $P_{\omega\pm}^{(\mathrm{NL})}$, the coupled mode equations (9) can be numerically integrated. However, due to the intrinsically complex algebraical form of the coupling coefficients, a qualitative understanding of the magneto-optical effects is aided by considering the linear case, with small nonlinear effects. For such a case, the coupled mode equations can be solved by a perturbative approach, to yield general integral solutions, as will now be outlined.

For the sake of algebraic simplicity, we define the effective coupling coefficient over the entire grating structure, from $z = 0$ to $z = L$, as

$$\kappa_{\mathrm{eff}}^{\pm}(z) = \kappa_{\mathrm{ed}}(z)\exp(i\beta_{\mathrm{ed}}^{\pm}z) \mp \kappa_{\mathrm{md}}(z)\exp(i\beta_{\mathrm{md}}^{\pm}z)\,,$$

including any amplitude and phase modulations of the linear optical and magneto-optical coefficients.

By spatially integrating Eq. (9b) from $z = 0$ to $z = L$, and using the boundary condition that no light is input at the end of the grating, $A^{\mathrm{b}}_{\mp}(L, \omega) = 0$, one obtains an integral expression for $A^{\mathrm{b}}_{\mp}(0, \omega)$ in terms of the spatial distribution of the forward propagating modes. On the other hand, the forward propagating modes can be expressed as integrals involving the backward propagating modes, by spatially integrating Eq. (9a). Inserting this expression into the integrated form of Eq. (9b) then yields

$$A^{\mathrm{b}}_{\mp}(0)/A^{\mathrm{f}}_{\pm}(0) = -\int_0^L \kappa^{\pm *}_{\mathrm{eff}}(z)\, dz - \frac{1}{A^{\mathrm{f}}_{\pm}(0)} \int_{z=0}^{L} \int_{z'=0}^{z} \kappa^{\pm *}_{\mathrm{eff}}(z) \kappa^{\pm}_{\mathrm{eff}}(z') A^{\mathrm{b}}_{\mp}(z')\, dz'\, dz\,. \tag{10}$$

In this expression, the first term on the right-hand side essentially comprises the superpositioned Fourier transforms of the optical and magneto-optical coupling coefficients $\kappa_{\mathrm{ed}}(z)$ and $\kappa_{\mathrm{md}}(z)$, windowed in the spatial domain by a unitary step function over the extent of the grating. In this transform interpretation, the detuning coefficients $\beta^{\pm}_{\mathrm{ed}}$ and $\beta^{\pm}_{\mathrm{md}}$ have the roles of spatial frequencies of the grating. The remaining term in the right-hand side of Eq. (10) is given in terms of the spatial distribution of the backward propagating modes, and for a sufficiently weak backward scattering, this term can be neglected since it involves a second-order dependence of the coupling coefficient.

Whenever the product of the coupling coefficient and the grating length cannot be considered as small, the integral forms of Eqs. (9) can be repeatedly applied, to give an expression for the reflected light in terms of the following generalized perturbation series, where the nth term is of the order of the coupling coefficient to the power of $2n - 1$,

$$\begin{aligned} A^{\mathrm{b}}_{\mp}(0)/A^{\mathrm{f}}_{\pm}(0) &= -\int_0^L \kappa^{\pm *}_{\mathrm{eff}}(z)\, dz + \int_{z_1=0}^{L} \int_{z_2=0}^{z_1} \int_{z_3=z_2}^{L} \kappa^{\pm *}_{\mathrm{eff}}(z_1) \kappa^{\pm}_{\mathrm{eff}}(z_2) \kappa^{\pm *}_{\mathrm{eff}}(z_3)\, dz_3\, dz_2\, dz_1 \\ &\quad + \cdots + (-1)^n \int_{z_1=0}^{L} \int_{z_2=0}^{z_1} \int_{z_3=z_2}^{L} \int_{z_4=0}^{z_3} \int_{z_5=z_4}^{L} \cdots \int_{z_{(2n-2)}=0}^{z_{(2n-3)}} \int_{z_{(2n-1)}=z_{(2n-2)}}^{L} \\ &\quad \times \kappa^{\pm *}_{\mathrm{eff}}(z_1) \kappa^{\pm}_{\mathrm{eff}}(z_2) \kappa^{\pm *}_{\mathrm{eff}}(z_3) \cdots \kappa^{\pm}_{\mathrm{eff}}(z_{(2n-2)}) \kappa^{\pm *}_{\mathrm{eff}}(z_{(2n-1)}) \\ &\quad \times dz_{(2n-1)}\, dz_{(2n-2)} \cdots dz_3\, dz_2\, dz_1 \\ &\quad + O[(\kappa^{\pm}_{\mathrm{eff}})^{2n}]\,. \end{aligned} \tag{11}$$

By applying Eq. (11) to an arbitrary level of precision, the integral solutions give the spectral response of an arbitrary spatial distribution of the refractive index and gyration constant.

The above described direct scattering approach mainly applies to linear magneto-photonic structures, with the coupling coefficients being independent of the intensity of the optical field. However, for qualitative arguments, the refractive index $n(z)$ and gyration constant $g(z)$, as they appear in the coupling coefficients and detuning parameters, can be replaced by their field-corrected forms $n(I, z)$ and $g(I, z)$. By using the field-corrected forms, however, the perturbation series (11) depends on the spatial, intra-grating intensity distribution, which *a priori* is not known. By repeatedly applying the perturbation series, with the intensity distribution in the linear optical regime as initial profile, one can iteratively obtain the full description of the light scattering and spectral response in the nonlinear optical regime as well.

7. Transfer Matrix Formulation

The direct scattering method for solving the coupled mode equations (9) can be considered as a semi-numerical approach, since it is applicable not only to numerical simulations, but also to finding analytical solutions as well. For many applications, however, the method of the transfer matrix formalism is preferable, such as in cases where the structure of interest is composed of a number of locally homogeneous layers.

In the transfer matrix formulation, the magneto-photonic device is discretized into a finite number of locally homogeneous layers, over which known solutions to the homogeneous wave equation are applied for wave propagation. This stacked-layer model is schematically shown in Fig. 2. As a benefit of the transfer matrix formalism, in contrast to the series expansion of the direct scattering approach, gratings of arbitrary strength can be analyzed, and multiple scattering effects that occur inside the grating are explicitly included throughout the analysis. For a sufficiently large number of thin, locally homogeneous layers, the discretized model provided by the transfer matrix is also applicable to continuously modulated distributions of the index of refraction and magneto-optical gyration constant.

The transfer matrix formalism involves products of two classes of matrices: one class for the wave propagation across the locally homogeneous layers in the model of the device, and one class for the infinitesimal transition over plane interfaces between different layers. We construct a four-dimensional vector

$$\mathbb{E}_k(z) = \left(E^{\mathrm{f}}_{k_+}(z) E^{\mathrm{f}}_{k_-}(z) E^{\mathrm{b}}_{k_+}(z) E^{\mathrm{b}}_{k_-}(z) \right)^{\mathrm{T}}$$

for the local optical fields inside the kth layer of the structure, defined over $z_k < z < z_{k+1}$, $k = 1, 2, \ldots, N-1$. We also define $\mathbb{E}_0$ and $\mathbb{E}_N$ to be the corresponding incident and transmitted fields, taken immediately to the left and right of the first and Nth layer interfaces, respectively.

The optical fields immediately to the left and right of the kth interface, at $z = z_k^-$ and $z = z_k^+$ respectively, are then related through

$$\mathbb{E}_k(z_k^+) = \mathbb{T}_k \mathbb{E}_{k-1}(z_k^-)\,, \tag{12}$$

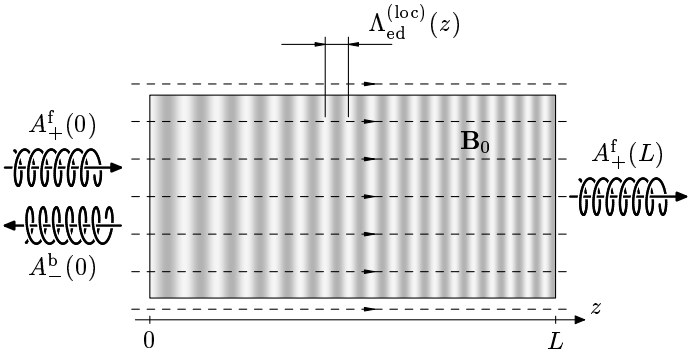

Fig. 1. Schematic figure of a continuous refractive index distribution $n(z)$ of the stratified magneto-photonic crystal.

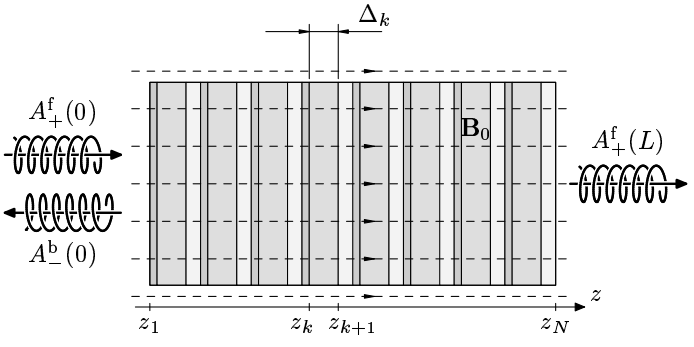

Fig. 2. Schematic figure of the stacked model structure as used in the transfer matrix formulation. The model comprises N interfaces in a total of $N-1$ locally homogeneous magneto-optical layers.

where the transfer matrix governing the field transition over the interface is given as

$$\mathbb{T}_k = \begin{pmatrix} 1/\tau'_{k_-} & 0 & 0 & \rho'_{k_-}/\tau'_{k_-} \\ 0 & 1/\tau'_{k_+} & \rho'_{k_+}/\tau'_{k_+} & 0 \\ 0 & -\rho_{k_-}/\tau'_{k_+} & 1/\tau'_{k_+} & 0 \\ -\rho_{k_+}/\tau'_{k_-} & 0 & 0 & 1/\tau'_{k_-} \end{pmatrix},$$

with elements given in terms of the reflection and transmission coefficients, including linear magneto-optical interactions,

$$\tau_{k_\pm} = \frac{2(n_{k-1} \pm g'_{k-1})}{n_{k-1} + n_k \pm (g'_{k-1} + g'_k)},$$

$$\tau'_{k_\pm} = \frac{2(n_k \mp g'_k)}{n_{k-1} + n_k \mp (g'_{k-1} + g'_k)},$$

$$\rho_{k_\pm} = \frac{n_{k-1} - n_k \pm (g'_{k-1} - g'_k)}{n_{k-1} + n_k \pm (g'_{k-1} + g'_k)} = -\rho'_{k_\mp},$$

with the notation $g'_k = -g_k/2n_k$. Here, $\tau_{k_\pm}$ are the amplitude transmission coefficients for left/right circularly-polarized waves propagating in the positive z-direction, while $\rho_{k_\pm}$ are the analogous amplitude reflection coefficients of the same wave components. Primed quantities are similarly related to the field quantities of opposite direction of propagation.

As a signature of the non-absorptive interface conditions between the layers, the reflection and transmission coefficients of the elements of $\mathbb{T}_k$ are related through the Stokes relations[11] $\rho_{k_\pm} = -\rho'_{k_\mp}$ and $\tau_{k_\pm}\tau'_{k_\mp} = 1 - \rho^2_{k_\pm}$, which were also used in the formulation of Eq. (12).

The relation between the optical field components in one end of a layer and the components in the other end of the layer is given as

$$\mathbb{E}_k(z^-_{k+1}) = \mathbb{P}_k\mathbb{E}_k(z^+_k)\,, \tag{13}$$

where

$$\mathbb{P}_k = \exp[i(\omega\Delta_k/c)]\exp[\mathrm{diag}(n_{k+}, n_{k-}, -n_{k-}, -n_{k+})]$$

is the $[4\times 4]$ exponential eikonal matrix, governing the wave propagation in the kth locally homogeneous layer of the structure. In the matrix $\mathbb{P}_k$, $n_{k\pm} = n_k(1 \mp g_k/2n_k^2)$ are the refractive indices experienced by circularly-polarized optical fields in the kth layer, and $\Delta_k = z_{k+1} - z_k$ are the corresponding layer thicknesses. In Eqs. (12) and (13), the experienced refractive indices $n_{k\pm}$ are allowed to be complex-valued quantities $n_{k\pm} = n'_{k\pm} + in''_{k\pm}$, where positive imaginary parts $n''_{k\pm}$ correspond to energy absorption in the medium.

By subsequently applying Eqs. (12) and (13), the incident and transmitted optical fields of the magneto-optical Bragg grating are then related through

$$\mathbb{E}_N = \widetilde{\mathbb{T}}\mathbb{E}_0\,, \tag{14}$$

where

$$\widetilde{\mathbb{T}} = \mathbb{T}_N\mathbb{P}_{N-1}\mathbb{T}_{N-1}\mathbb{P}_{N-2}\cdots\mathbb{P}_1\mathbb{T}_1$$

is the total transfer matrix of the compound magneto-photonic structure.

As in the direct scattering approach, the above described method applies mainly to linear magneto-photonic structures, for which the state of all of the partial transfer matrices $\mathbb{P}_k$ and $\mathbb{T}_k$ are independent of the state of light. Whenever the refractive index and gyration constant are replaced by their field-corrected quantities, the partial transfer matrices become dependent on the local intensity and ellipticity of the light, and in particular the calculation of the phase-affecting propagation matrices $\mathbb{P}_k(I)$ becomes crucial.

Since one *a priori* does not know the intra-grating spatial distribution of the light intensity and polarization state, one cannot in general start the analysis at the incoupling end of the grating. Due to the inherently multivalued nature of nonlinear optical devices with optical feedback, and also due to the fact that the backward propagating light at the end of the grating is zero, the transfer matrix method is

instead straightforwardly applied to the inverse problem, that is to say, the problem of calculating the input light wave that corresponds to a given transmitted wave. In this case, the transfer matrix method yields an inverse formulation of Eq. (14), where we first of all initialize the transmitted field vector

$$\mathbb{E}_N = \left(E^{\mathrm{f}}_{N_+}(z_N^+) \;\; E^{\mathrm{f}}_{N_-}(z_N^+) \;\; 0 \;\; 0 \right)^{\mathrm{T}} .$$

Using this vector as initial parameter, the field in the $(N-1)$th layer can then be calculated by first using the inverse formulation of Eq. (12) as

$$\mathbb{E}_{N-1}(z_N^-) = \mathbb{T}_N^{-1} \mathbb{E}_N(z_N^+) \,, \tag{15}$$

which for non-absorbing locally homogeneous layers gives the intensity I_{N-1} and ellipticity ϵ_{N-1} of the polarization state of the light in the $(N-1)$th layer. In Eq. (15), the reflection coefficients involved in the elements of $\mathbb{T}_N^{-1}$ can be calculated using the linear refractive indices and gyration constants. Using the information of the intensity and ellipticity, the light wave at the beginning of the $(N-1)$th layer can then be calculated using the inverse and field-corrected form of Eq. (13),

$$\mathbb{E}_{N-1}(z_{N-1}^+) = \mathbb{P}_{N-1}^{-1}(I_{N-1}, \epsilon_{N-1}) \mathbb{E}_{N-1}(z_N^-) \,. \tag{16}$$

By subsequently applying Eqs. (15) and (16), we can finally calculate the initial field $\mathbb{E}_0$ that corresponds to the given transmitted field $\mathbb{E}_N$. This formulation of the inverse problem provides a powerful tool that is perfectly adequate for qualitative as well as quantitative analysis of an implementation of a certain design of a magneto-photonic structure.

8. Binary Structures

For grating structures composed of locally homogeneous layers of altering optical and magneto-optical coefficients, denoted here as binary magneto-photonic structures, the design of a setup is considerably simplified, since it allows us to apply a well-known analytical theory and arguments of thin-film theory.[24]

The design procedure is illustrated here with a binary structure consisting of two stacked distributed feedback structures, each one comprising layers of altering refractive index $n_{\mathrm{l}} = 1.8$ and $n_{\mathrm{h}} = 2.4$. In this example, the high index medium also possesses non-zero magneto-optical coefficients. The two stacks enclose a centre layer, also denoted as the "defect" layer, which for simplicity, is also composed of the high index medium. The spatial index profile of the compound structure is shown in Fig. 3. Working in resonant reflection, using the terminology as introduced in the transfer matrix formalism of the previous section, the alternating layers are designed with respective layer thicknesses of $\Delta_k = \lambda_{\mathrm{c}}/4n_k$, where $\lambda_{\mathrm{c}} = 900$ nm is the design resonance vacuum wavelength in the absence of the static magnetic field. The peak reflectance of such a structure in free space, with M pairs of stacked layers and with magnetic field switched off, is[24]

$$R = \left[\frac{1 - (n_{\mathrm{h}}/n_{\mathrm{l}})^{2M}}{1 + (n_{\mathrm{h}}/n_{\mathrm{l}})^{2M}} \right]^2 ,$$

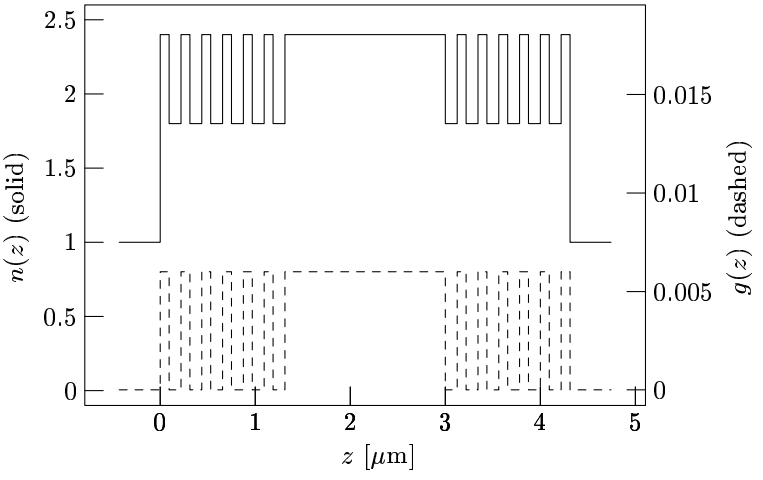

Fig. 3. Spatial modulation of linear refractive index $n(z)$ and gyration constant $g(z)$, for a structure consisting of $N-1=25$ alternating layers with indices $n_{\mathrm{h}}=2.4$ and $n_{\mathrm{l}}=1.8$. The structure is optimized for resonance at $\lambda_{\mathrm{c}}=900$ nm vacuum wavelength, with a center layer thickness of $9\lambda_{\mathrm{c}}/2n_{\mathrm{h}}$, and with surrounding mirror structures composed of alternating quarter-wavelength layers. The surrounding stacks of six layer pairs each provide an effective maximum reflectance of about 88.1 percent.

and for a moderate number of $M=6$ pairs, each of the enclosing stacks have respective peak reflectances of approximately 88.1 percent. In order to create a composite structure with narrow transmission peaks, the center layer is designed to be resonant at the same centre wavelength λ_{c} at which the surrounding binary stacks are resonant in reflection. The resonance condition for the center layer of thickness Δ_{13} yields $2\pi n_{\mathrm{h}}\Delta_{13}/\lambda_{\mathrm{c}}=m\pi$, and we choose the resonance order $m=9$, hence giving a center layer thickness of $\Delta_{13}=9\lambda_{\mathrm{c}}/2n_{\mathrm{h}}$. In total, this gives an overall thickness of $L=4.31$ μm of the structure.

In the linear optical regime this structure will have a unity maximum transmittance at exact cavity resonances, since the enclosing stacks are balanced with an equal reflectance. In the spectral vicinity of the center wavelength, for which the reflectance of the surrounding stacks is still high, the minimum transmission can be estimated from classical Fabry–Pérot theory to yield $T_{\min}=(1-R)^2/(1+R)^2\approx 0.0040$, hence corresponding to an extinction ratio of $-10\log(T_{\min}/T_{\max})\approx 24$ dB between maximum and minimum transmittance. The transmittance as a function of vacuum wavelength is shown in Fig. 4, which was generated using the transfer matrix formalism as described in Sec. 7.

Expressed in terms of the refractive indices experienced by the circularly-polarized modes, the Faraday rotation angle, measured clockwise around the axis of the direction of propagation of the light, is

$$\Phi_{\mathrm{F}}=\frac{\omega}{2c}(n_+-n_-)=\frac{\omega g_0}{2n_0 c}\,.$$

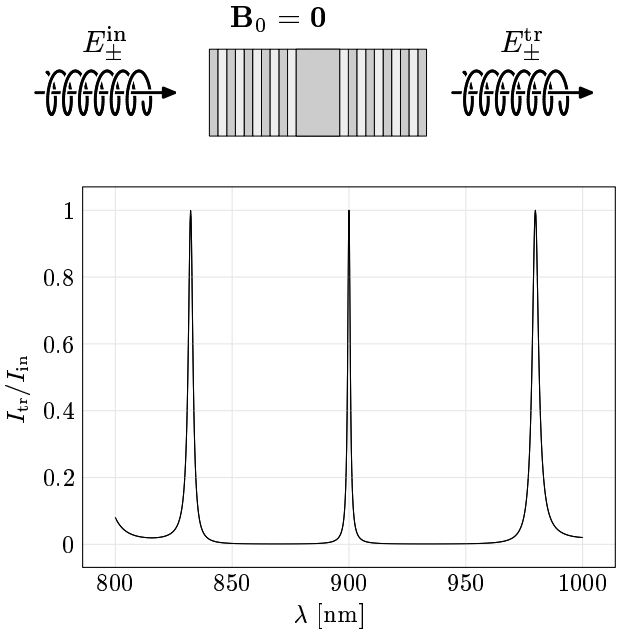

Fig. 4. Transmission characteristics of the magneto-optical Bragg grating as shown in Fig. 3, with the magnetic field switched off.

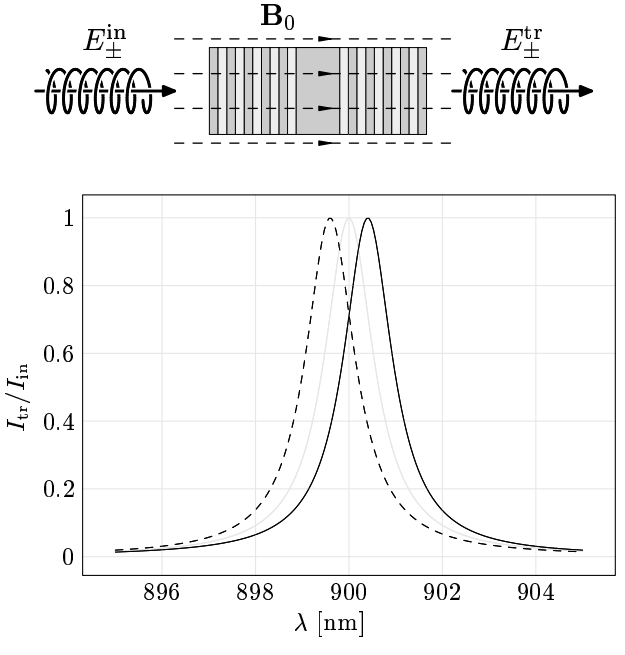

Fig. 5. Transmission characteristics of the same magneto-optical Bragg grating as in Fig. 4, but with the magnetic field switched on. Solid/dashed lines show the transmission for left/right circularly-polarized input light.

For a modest value of $\Phi_{\mathrm{F}} = 0.5$ deg/μm around λ_{c}, this gives a value of the gyration constant as $g_0 = 6.0 \times 10^{-3}$. In terms of the differential refractive index change experienced by circularly-polarized modes inside the high index medium, this corresponds to $n_{\pm} - n_{\mathrm{h}} = \mp 2.6 \times 10^{-4}$. This differential change in refractive index has its main impact in a differential change of the Bragg resonances of the structure, with the transmission peaks of right circular polarization shifting to the right in the spectrum, while those of left circular polarization experiencing the same shift but in the opposite direction. This lifting of the degeneracy of the Bragg resonances is shown in Fig. 5, which was also generated using the transfer matrix formalism.

Whenever the surrounding stacked layers around the center one possess non-zero magneto-optical coefficients, the exact displacement of the resonance peaks of the structure generally must be calculated using, for example, the transfer matrix formalism. In particular, the non-reciprocity causes a slight increase in index contrast with accompanying increase in reflection for one of the circularly-polarized modes, while the orthogonal mode simultaneously experiences a decrease. However, whenever the cavity resonance of the center layer is comparatively strong, as compared to the distributed resonances of the surrounding stacks, one may estimate the displacement of the center resonance from classical Fabry–Pérot theory as

$$\Delta\lambda_{\mathrm{c}} \approx g_0 \lambda_{\mathrm{c}} / 2n_0^2 \,.$$

With the previously chosen parameters, an estimation of the differential shift of the resonance peaks of left and right circular polarizations is $\Delta\lambda_{\mathrm{c}} = 0.47$ nm, or a peak separation of 0.94 nm.

For moderate magneto-optical coefficients, the peak separation is comparatively small; however, for structures possessing a sufficiently high cavity finesse, for which the half maximum full-width of the peaks are small, the small change in Bragg resonance has a dramatic effect on the intra-grating intensity distribution, as shown in Fig. 6.

Finally, we will make a few notes on the nonlinear behavior of the device under influence of strong optical fields. As an explicit signature of the non-reciprocity of the medium, nonlinear magneto-optical Fabry–Pérot cavities are asymmetrical in their ellipticity dependence of transmission.[15] In particular, by choosing a vacuum wavelength of operation in such a way that one of the circular polarization states is suppressed, a boosting of the asymmetrical behavior appears in a nonlinear optical regime.

The interpretation of the polarization state dependence of transmission properties is aided by the introduction of the Stokes parameters,[11] which for the input optical field yield

$$S_0 = [|E_+^{\mathrm{f}}|^2 + |E_-^{\mathrm{f}}|^2]_{z=0}, \quad S_1 = 2\,\mathrm{Re}[E_+^{\mathrm{f}*} E_-^{\mathrm{f}}]_{z=0}\,,$$
$$S_3 = [|E_+^{\mathrm{f}}|^2 - |E_-^{\mathrm{f}}|^2]_{z=0}, \quad S_2 = 2\,\mathrm{Im}[E_+^{\mathrm{f}*} E_-^{\mathrm{f}}]_{z=0}\,,$$

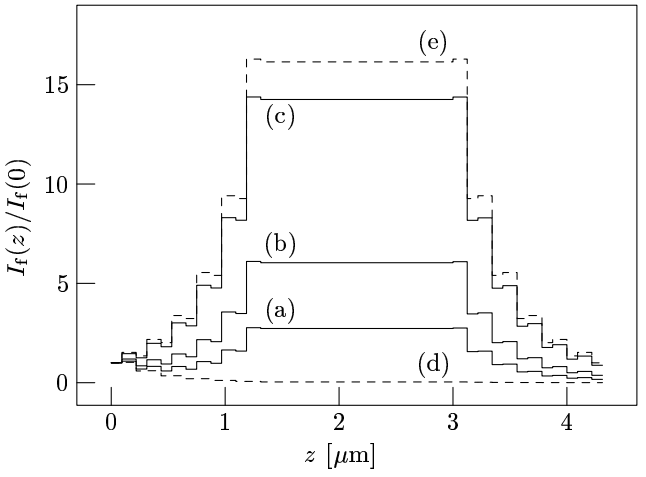

Fig. 6. The intra-grating intensity evolution of left circularly-polarized forward propagating modes inside the magneto-photonic structure, $I_{\rm f} = n(z)\varepsilon_0 c|A^{\rm f}_+(z)|^2/2$, with the static magnetic field switched on. The curves are calculated for vacuum wavelengths of (a) $\lambda_0 = 898.0$ nm, (b) $\lambda_0 = 899.0$ nm, and (c) $\lambda_0 = 900.0$ nm (solid curves). The dashed curves (d) and (e) show the corresponding intensity evolution at minimum and maximum transmittance, respectively.

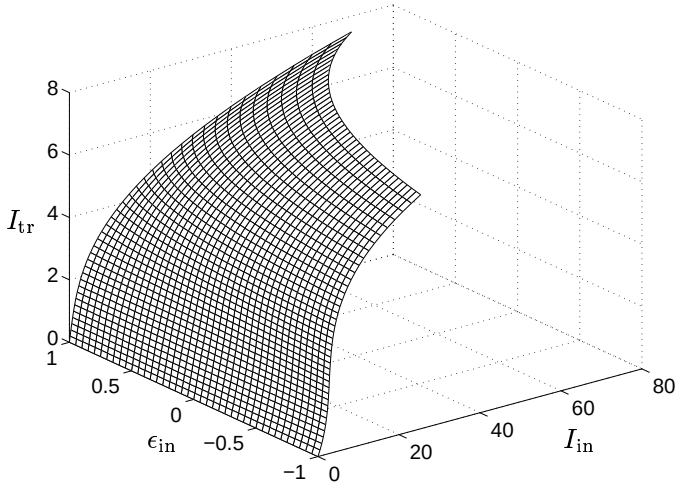

Fig. 7. Transmitted intensity $I_{\rm tr} = \varepsilon_0 c W_0/2$ as a function of input intensity $I_{\rm in} = \varepsilon_0 c S_0/2$ and normalized ellipticity $\epsilon_{\rm in} = S_3/S_0$, evaluated at a vacuum wavelength of $\lambda_0 = 900.55$ nm.

and similarly for the transmitted field

$$W_0 = [|E^{\rm f}_+|^2 + |E^{\rm f}_-|^2]_{z=L}, \quad W_1 = 2\,{\rm Re}[E^{\rm f*}_+ E^{\rm f}_-]_{z=L}\,,$$
$$W_3 = [|E^{\rm f}_+|^2 - |E^{\rm f}_-|^2]_{z=L}, \quad W_2 = 2\,{\rm Im}[E^{\rm f*}_+ E^{\rm f}_-]_{z=L}\,,$$

where $E^f_\pm = \mathbf{e}^*_\pm \cdot \mathbf{E}^{\rm f}_\omega$ are the forward propagating components of the electric field $\mathbf{E}_\omega$.

In Fig. 7, the transmitted intensity $I_{\rm tr} = \varepsilon_0 c W_0/2$ is shown as a function of the input intensity $I_{\rm in} = \varepsilon_0 c S_0/2$ and the input normalized ellipticity $\epsilon_{\rm in} = S_3/S_0$. In the figure, the normalized input ellipticity ranges from $\epsilon_{\rm in} = -1$ (right circular polarization) via $\epsilon_{\rm in} = 0$ (linear polarization) to $\epsilon_{\rm in} = +1$ (left circular polarization), while the intensity ranges up to an input intensity corresponding to a photo-induced refractive index change of maximum

$$\delta n = \frac{3}{8n_0}\chi^{(\rm eeee)}_{xxxx}|\mathbf{E}_\omega|^2 = 1.40 \times 10^{-4}\,.$$

In the figure, the asymmetrical impact of the non-reciprocity on the transmission is clearly present, and as the analysis is extended to more confined cavity effects, for structures with an even higher finesse, optically multistable solutions appear as well.

9. Conclusions

In this paper, we have focused on a review of the theoretical tools and fundamental concepts of magneto-photonic structures. Material issues and configurations were discussed, and the fundamental concepts of photo-induced modification of linear optical and magneto-optical properties of matter were reviewed. The coupled mode equations governing wave propagation inside magneto-photonic structures were derived from a first-principles approach, and their solution in terms of a general perturbation series, yielding integral solutions, was described. Moreover, the transfer matrix formalism as applied to these structures was reviewed, and issues of the method of solving the inverse problem of nonlinear magneto-photonic crystals was discussed. Finally, as an illustration of the application of the theoretical tools discussed, a binary structure with a center defect layer was analyzed in transmittance by means of the transfer matrix formalism.

The key role played by the non-reciprocity inherent in these magneto-photonic structures is strikingly evidenced in their distinctly different characteristics and behavior with respect to those of the photonic ones; thus these are different for left and right circularly-polarized light beams and also for forward and backward propagating beams of same circular polarization state. This indicates that these structures could efficiently be used for circular polarization state discrimination and filtering, spatial direction recognition and velocity sensitivity along with enhancing the performances of other more straightforward applications such as isolators, modulators and sensors. With the enhancement of the field intensity because of the confinement in these structures we also expect the nonlinear regime to become accessible and lead to photoinduced modification and control of their characteristics.

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