Thank you! The following message has been sent to the *MathWorld* team:

Source URL: http://mathworld.wolfram.com/EllipticIntegral.html

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Comment type: TYPO

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Comments: Hi Eric, I am writing to alert you on a few errors which I believe are present in the chapter on elliptic integrals [http://mathworld.wolfram.com/EllipticIntegral.html], in the section describing the Weierstrass approach, Eqs. (51)-(56). For the sake of simplicity, I will as a shorthand notation below denote this chapter by "EI", with equations written in plain TeX style. First of all, the solution given by EI Eq. (56) is only valid provided that EI Eq. (51) is reformulated in the context of the integral equation $$ \int^{v(\zeta)}_{v(\zeta_0)}{{dx} \over{({a_4x^4+4a_3x^3+6a_2x^2+4a_1x+a_0})^{1/2}}}=\zeta, $$ that is to say, with the quartic $f(x)$ of the form $f(x)=a_4x^4+4a_3x^3+6a_2x^2+4a_1x+a_0$, rather than the form as given in EI Eq. (52). In addition, the integral given by EI Eq. (51) looks somewhat odd as the variable of integration x also appear as a limit. The integral equation given above then has the explicit solution in terms of the Weierstrass elliptic function as $$ v(\zeta)=v_0+ {{\sqrt{f(v_0)}\wp'(\zeta)+{\textstyle{{1}\over{2}}}f'(v_0) [\wp(\zeta)-{\textstyle{{1}\over{24}}}f''(v_0)] +{\textstyle{{1}\over{24}}}f(v_0)f'''(v_0)} \over{2[\wp(\zeta)-{\textstyle{{1}\over{24}}}f''(v_0)]^2 - {\textstyle{{1}\over{48}}}f(v_0)f^{\rm (iv)}(v_0)}}. $$ with quartic invariants $$ g_2\equiv a_0a_4-4a_1a_3+3a^2_2,\qquad g_3\equiv \left\vert\matrix{a_0&a_1&a_2\cr a_1&a_2&a_3\cr a_2&a_3&a_4\cr}\right\vert =a_0a_2a_4+2a_1a_2a_3-a^3_2-a_0a^2_3-a^2_1a_4. $$ Here it should be noticed that the invariant g_3 given in EI Eq. (54) is wrong, while the form of the solution EI Eq. (56) still is correct, provided that the integral equation as displayed above is used instead. That the solution listed above actually *is* a solution is easily verified with the following MapleV blocks: restart: f:=a[0]*v^4+4*a[1]*v^3+6*a[2]*v^2+4*a[3]*v+a[4]; g[2]:=a[0]*a[4]-4*a[1]*a[3]+3*a[2]^2; g[3]:=a[0]*a[2]*a[4]+2*a[1]*a[2]*a[3]-a[2]^3-a[0]*a[3]^2-a[1]^2*a[4]; df[0]:=eval(f,v=v0): df[1]:=eval(diff(f,v$1),v=v0): df[2]:=eval(diff(f,v$2),v=v0): df[3]:=eval(diff(f,v$3),v=v0): df[4]:=eval(diff(f,v$4),v=v0): tmp[1]:=sqrt(df[0])*WeierstrassPPrime(z,g[2],g[3]): tmp[2]:=(1/2)*df[1]*(WeierstrassP(z,g[2],g[3])-(1/24)*df[2]): tmp[3]:=(1/24)*df[0]*df[3]: tmp[4]:=2*(WeierstrassP(z,g[2],g[3])-(1/24)*df[2])^2: tmp[5]:=(1/48)*df[0]*df[4]: v:=v0+(tmp[1]+tmp[2]+tmp[3])/(tmp[4]-tmp[5]): p:=a[0]*v^4+4*a[1]*v^3+6*a[2]*v^2+4*a[3]*v+a[4]: testfunc:=(diff(v,z))^2-p: testfunc:=simplify(testfunc); Best regards, Fredrik Jonsson, Southampton, England

***MathWorld* usage:** occasionally

***Mathematica* usage:** never